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A 2DOF state feedback MRAC control of an electromechanical system*

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***Abstract*— In this paper, a real-time state feedback Model Reference Adaptive Control strategy is developed for stabilization of a 3DOF Hover electromechanical system, and its disturbance rejection properties are analyzed.**

***Index Terms*— Model Reference Adaptive Control (MRAC), 2DOF state feedback control, Real-time control, Electromechanical systems**

I. Introduction

Several approaches have been developed to design an adaptive controller, a controller with adjustable parameters and a mechanism for adjusting the parameters [1], to cope with changing plant dynamics. They are usually classified as direct adaptive control, where a control design method is combined with the control parameter adjust mechanism, and indirect adaptive control, where the control design method is combined with a plant parameter identification mechanism.

Applying adaptive techniques to the control of aerial unmanned vehicles has been a topic of increasing research interest, as adaptive control is a non linear control strategy well suited to approach this kind of specific plant structure. Different adaptive control design strategies have been proposed, like R,S,T two degree of freedom polynomial controller design methods, fuzzy controllers, backstepping [2], [3], [4], [5], [6].

In this paper, a Lyapunov based direct model reference adaptive control is proposed for both plant input disturbance rejection and real-time tuning of a two degree state feedback control. The open-loop plant may be stable or unstable, but without zeros. This is a model frequently used when approximately modeling plants, suitable for unmanned aerial vehicles (UAV), [7], where many parameters are negligible and the model is reduced to a chain of integrators. The design is proved to stabilize an unstable electromechanical system, showing good simulation and experimental results.

The paper is structured as follows. In section II the proposed design method is presented. Section III describes the experimental platform utilized to illustrate the MRAC design method. In section IV some experimental results are obtained, being discussed in section V as conclusions.

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II. A two degree of freedom state feedback MRAC

A model reference adaptive control strategy combines a fast regulatory loop with a slow parameter tuning loop. In this case, a continuous time two degree of freedom state feedback controller design is considered, as shown in Fig. 1.

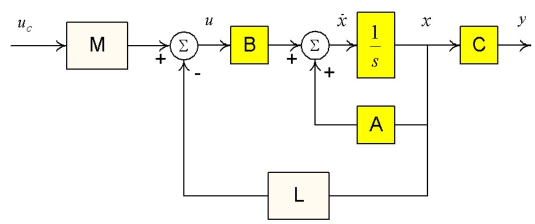

Fig. 1. Continuous time 2DOF state feedback control.

Consider a SISO plant (1) with input u and output y

$$y(s) = G(s)u(s); \quad \rightarrow G(s) = \frac{b}{a(s)} \tag{1}$$

where

$$a(s) = a_1 + a_2 s + \ldots + a_n s^{n-1} + s^n \tag{2}$$

and $\frac{b}{a_1}$ is the static gain of the plant, if stable. For this system, the controllable canonical state space representation (A,B,C) is given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{3}$$

where $x \in R^n$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & \ldots & b \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}$$

(and $D=0$, non direct output/input coupling). Assume a plant model reference (4) with input u_c and output $\overline{y}$

$$\overline{y}(s)=\overline{G}(s)u_c(s);\quad \rightarrow \overline{G}(s)=\frac{\overline{b}}{\overline{a}(s)} \tag{4}$$

with characteristic polynomial

$$\overline{a}(s)=\overline{a}_1+\overline{a}_2 s+\ldots+\overline{a}_n s^{n-1}+s^n \tag{5}$$

The internal representation $(\overline{A},\overline{B},\overline{C})$ of the model reference (4), using the same structure that (3) is obtained with $\overline{a}_i$ instead of a_i, $\overline{b}$ instead of b and state vector $\overline{x}$.

For systems with no zeros[1] perfect tracking of the model reference state vector $\overline{x}$ can be obtained by using a two degree of freedom (2DOF) state feedback controller

$$u=Mu_c-Lx \tag{6}$$

where M is a scalar and $L\in R^n$ is a row vector. From (3) and (6), the state space closed loop dynamics is given by

$$\dot{x}=(A-BL)x+BMu_c=\overline{A}x+\overline{B}u_c \tag{7}$$

Thus, assuming full knowledge of the plant model (3), the ideal parameters L and M of (6) matching the closed loop model reference (4) are given by

$$\begin{aligned} L^* &= \begin{bmatrix} \frac{\overline{a}_1-a_1}{b} & \frac{\overline{a}_2-a_2}{b} & \cdots & \frac{\overline{a}_n-a_n}{b} \end{bmatrix} \\ M^* &= \frac{\overline{b}}{b} \end{aligned} \tag{8}$$

yielding

$$\overline{A}=A-BL^*;\quad \overline{B}=BM^* \tag{9}$$

Obviously, $\overline{A}$ defines the stable closed loop dynamics to follow, whereas $\overline{b}/\overline{a}_1$ defines the static gain of the reference model.

A. Adaptation law

Note that $\overline{a}_i$, $\overline{b}$ are design parameters and a_i, b are assumed to be known. But usually, the plant state matrix A as well as the parameter b representing the product of the actuator and output sensor gain can be only estimated. One approach to obtain model reference state perfect tracking with partial or without plant knowledge is to apply an adaptive strategy that automatically brings to zero the steady state tracking error. Here a Lyapunov based adaptive strategy is developed to tune the 2DOF state feedback controller (6) for unknown plant parameters. As will be shown later, the required knowledge is the plant gain sign as well as the accessibility to the plant state x, the model reference state $\overline{x}$ and the input command signal u_c.

The adaptive tuning goal is to obtain the (L,M) parameters dynamically for both, convergence to the "true" parameters and zero steady state tracking error vector, that is to obtain $L\rightarrow L^*$, $M\rightarrow M^*$ and $\lim_{t\rightarrow\infty}\|z(t)\|=0$, $\forall t\geq 0$ from any initial condition $\|z(0)\|\neq 0$, where $z=x-\overline{x}$ is the tracking error vector. This goal can be achieved by using the Lyapunov stability theory, as follows.

[1] It is well-known [8] that the plant poles can be assigned by state feedback but there is no action on the zeros position.

Let $V(z,K)$ be a Lyapunov candidate function,

$$V(z,K)=\frac{1}{2}\left(\gamma z^T Pz+(K-K^*)^T(K-K^*)\right) \tag{10}$$

where z is the previously defined tracking error vector, $K=[L\ M]^T$ is the parameter vector of dimension $(n+1)\times 1$, and P is a symmetric positive definite matrix ($P>0$). Clearly, the continuously differentiable candidate function (10) meets the Lyapunov necessary conditions: $V(z,K)>0\quad \forall z\neq 0, \forall K\neq K^*=[L^*\ M^*]^T$ and $V(0,K^*)=0$. In addition to these conditions, Lyapunov global asymptotically stability is assured if and only if $\dot{V}(z,K)<0$. In that case $z=0$ and K^* will be globally asymptotically stable equilibrium points.

The tracking error vector $z(t)$ dynamics can be expressed as (11)

$$\dot{z}=\dot{x}-\dot{\overline{x}}=(A-BL)x-\overline{A}\overline{x}+(BM-\overline{B})u_c \tag{11}$$

Adding and subtracting the term BL^*x and using (9), this equation can be arranged as

$$\begin{aligned} \dot{z} &= \overline{A}z+B[-(L-L^*)x+(M-M^*)u_c] \\ &= \overline{A}z+B[K-K^*]^T \begin{bmatrix} -x \\ u_c \end{bmatrix} \end{aligned} \tag{12}$$

Due to the special form of B, it can be simplified as

$$\dot{z}=\overline{A}z+\begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ & b[K-K^*]^T & \end{bmatrix}\begin{bmatrix} -x \\ u_c \end{bmatrix} \tag{13}$$

which is equivalent to

$$\begin{aligned} \dot{z} &= \overline{A}z+b\begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ & [-x^T \quad u_c] & \end{bmatrix}[K-K^*] \\ &= \overline{A}z+\Gamma[K-K^*] \end{aligned} \tag{14}$$

where $\Gamma=\Gamma(x,u_c)$ is a matrix of dimension $n\times(n+1)$

$$\Gamma=b\begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ & [-x^T \quad u_c] & \end{bmatrix} \tag{15}$$

with a gain being adapted in the estimation process $b=\overline{b}/M^*$.

Differentiating (10) and combining it with the error vector dynamics (14) yields to

$$\dot{V}(z,K)=\frac{1}{2}\gamma z^T(\overline{A}^T P+P\overline{A})z+(K-K^*)^T(\dot{K}+\gamma\Gamma^T Pz) \tag{16}$$

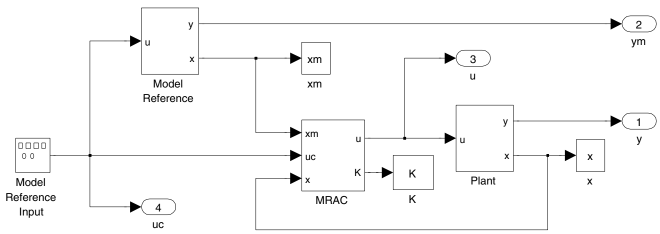

Fig. 2. 2DOF state space MRAC scheme.

From this result, two conditions must be fulfilled to obtain global asymptotic stability

i) $\overline{A}^T P + P\overline{A} = -Q$, for any given $Q > 0$

ii) $\dot{K} = -\gamma \Gamma^T P z$, for any given $\gamma > 0$ $(b > 0)$

Condition i), known as the Lyapunov equation, is always satisfied for a stable model reference, and condition ii) represents the parameter adaptation law which requires the recursive computation of the non square matrix Γ. With both conditions $\dot{V}(z,K) = -\frac{1}{2}\gamma z^T Q z < 0$ is obtained, where γ defines the convergence velocity in a trade-off with the control effort.

The proposed adaptive strategy is shown in Fig. 2

B. 2DOF MRAC simulation examples

Two examples are used to illustrate the performance of the proposed adaptive scheme. The first one considers an unstable third order plant and the second one is a double integrator SISO plant with input disturbances.

1) 2DOF adaptive control of an unstable third order plant: Let us consider an unstable third order plant with transfer function

$$G(s) = \frac{1}{s^3 + 2s^2 + s + 10}$$

and assume the desired trajectory be represented by

$$\overline{G}(s) = \frac{8}{(s+2)^3}$$

That is

$$\left.\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} \dot{\overline{x}} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -12 & -6 \end{bmatrix} \overline{x} + \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix} u_c \\ \overline{y} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \overline{x} \end{aligned}\right\} \tag{18}$$

Using the design method for the 2DOF state feedback controller (8), yields

$$K^* = \begin{bmatrix} L^* & M^* \end{bmatrix}^T = \begin{bmatrix} \begin{bmatrix} \frac{\overline{a}_1 - a_1}{b} & \frac{\overline{a}_2 - a_2}{b} & \frac{\overline{a}_3 - a_3}{b} \end{bmatrix} & \frac{\overline{b}}{b} \end{bmatrix}^T \tag{19}$$

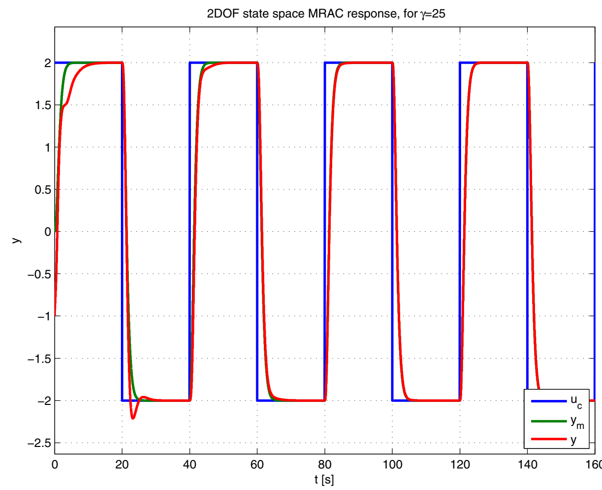

Fig. 3. 2DOF state space MRAC response.

That is, the elements of the nominal control law are $L_1^* = -2$; $L_2^* = 11$; $L_3^* = 4$; $M^* = 8$;.

Assuming full plant state accessibility, the tracking error vector dynamics can be obtained as $\dot{z} = \overline{A}z + \Gamma(K - K^*)$, where

$$\Gamma = b \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -x_1 & -x_2 & -x_3 & u_c \end{bmatrix} \tag{20}$$

As the state matrix $\overline{A}$ is stable, then $\exists P > 0$ solution to $\overline{A}^T P + P\overline{A} = -Q$ for any given $Q > 0$, for which the parameter adaptation law is obtained as $\dot{K} = -\gamma \Gamma^T P z$. Notice that the plant gain b in Γ should be estimated ($b = \overline{b}/M^*$) but it can be included into the selected adaptation gain γ. In this way the adaptation law just requires to know the sign of b, accessibility to the plant state x, the model reference state $\overline{x}$ and the input command signal u_c. The output error result is shown in Fig. 3 for $\overline{b} = 8$, $\overline{a}_1 = 8$, $\overline{a}_2 = 12$, $\overline{a}_3 = 6$ and $\gamma = 25$. The Lyapunov equation has been solved for $Q = I_{3\times 3}$, yielding to

$$P = \begin{bmatrix} 1.9063 & 1.2344 & 0.0625 \\ 1.2344 & 2.0938 & 0.1445 \\ 0.0625 & 0.1445 & 0.1074 \end{bmatrix}$$

The parameter adaptive law converges to the “true” values K^* with almost zero tracking error vector from $t = 100[s]$ onwards as shown in Fig. 4.

2) 2DOF adaptive control of a disturbed double integrator: Consider a double integrator plant with an input disturbance and the proposed stable model reference (18) with $b, \overline{b}, \overline{a}_1, \overline{a}_2 > 0$.

$$\begin{aligned} \dot{\overline{x}} &= \begin{bmatrix} 0 & 1 \\ -\overline{a}_1 & -\overline{a}_2 \end{bmatrix} \overline{x} + \begin{bmatrix} 0 \\ \overline{b} \end{bmatrix} u_c \\ \overline{y} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \overline{x} \end{aligned} \tag{21}$$

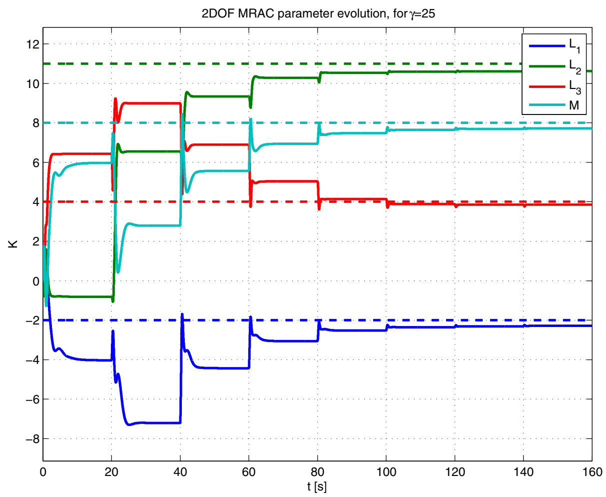

Fig. 4. Parameter error evolution.

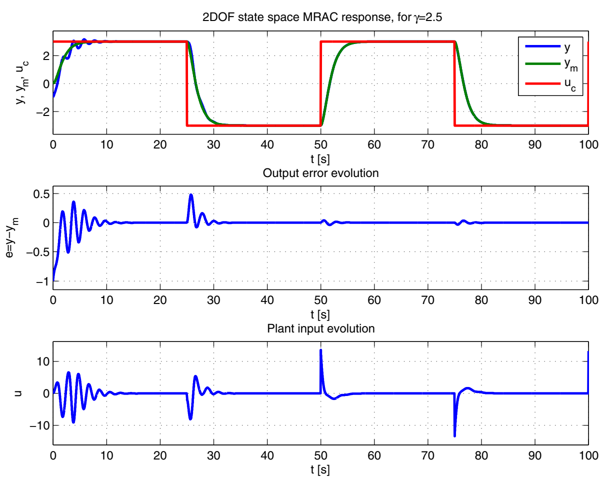

Fig. 5. 2DOF state space MRAC response.

Following the previous approach, the Lyapunov equation is solved for $Q = I_{2\times 2}$, yielding to

$$P = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

The output error result is shown in Fig. 5 for $\bar{b} = 1$, $\bar{a}_1 = 1$, $\bar{a}_2 = 2$ and $\gamma = 2.5$. The parameter adaptive law converges to the "true" values $K^* = [2\ 4\ 2]^T$ with zero tracking error vector from $t = 35[s]$ onwards as shown in Fig. 6.

Now, a periodic disturbance d_i signal is added to the plant input u almost without affecting the output error, as shown in Fig. 7. In this case, the plant state tends to the model reference state, but the parameters (L, M) are changed to compensate the periodic input disturbance, as shown in Fig. 8.

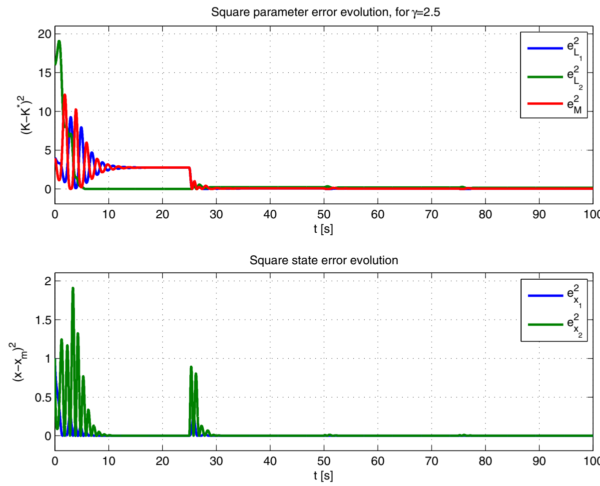

Fig. 6. Parameter and state error evolution.

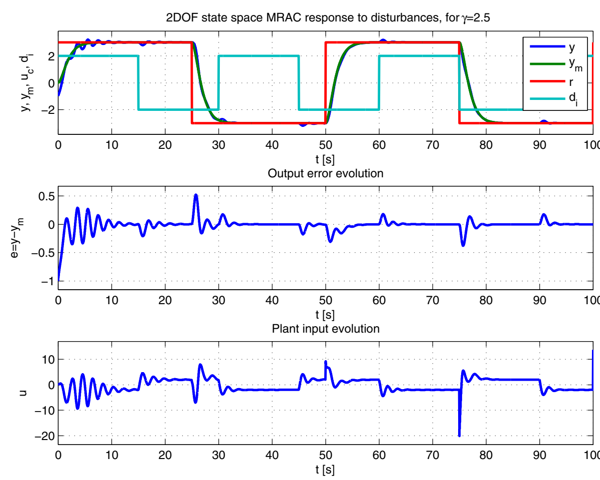

Fig. 7. 2DOF state space MRAC input disturbance response.

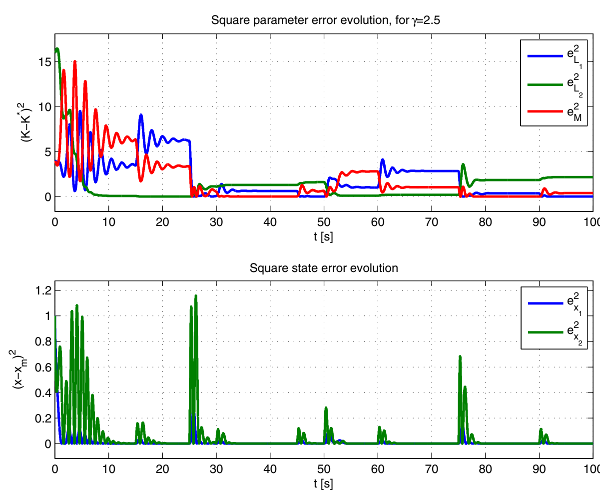

Fig. 8. Parameter and state error evolution.

III. The 3DOF Hover Experimental Platform

The experimental platform chosen to evaluate the performance of the designed MRAC control is a three degree of freedom (3DOF) system [9], [10]. The hardware platform, shown in Fig. 9, consists of a quadrotor mounted on a 3DOF pivot joint, such that the body can freely move in roll, pitch, and yaw angles. The data acquisition system of the platform is an Inertial Measurement Unit (3DM-GX2 [11]), measuring the position and angular velocity of the three orientation-axes of the quadrotor ϕ, θ, and ψ.

The control inputs are the voltages V_1, V_2, V_3, and V_4 applied to the 4 propellers of the quadrotor.

The non-linear dynamic model of this platform (22), as given in [12], is a result from approximating the Euler-Lagrange equations of motion on the (ϕ, θ, ψ) angular coordinates (roll, pitch and yaw angles)

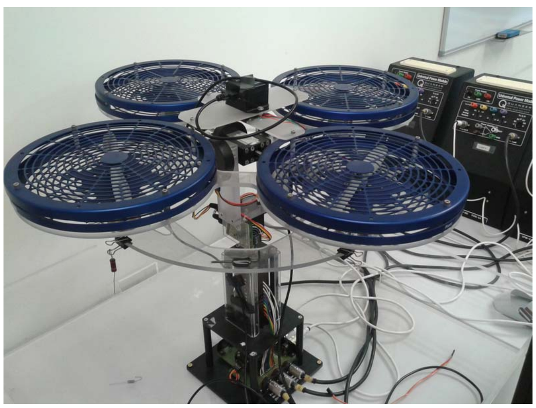

Fig. 9. 3DOF Hover Experimental Platform

$$\begin{aligned} \ddot{\phi} &= \frac{J_r}{I_{xx}}\dot{\theta}u_g + \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} + u_1 \\ \ddot{\theta} &= -\frac{J_r}{I_{xx}}\dot{\phi}u_g + \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\psi}\dot{\phi} + u_2 \\ \ddot{\psi} &= \frac{I_{xx}-I_{yy}}{I_{zz}}\dot{\theta}\dot{\phi} + u_3 \end{aligned} \quad (22)$$

Input u_g represents the gyroscopic effect in the roll and pitch dynamics, which is the sum of the (known) applied input voltages

$$u_g = K_v(V_1 + V_3 - V_2 - V_4) \quad (23)$$

and u_1, u_2, u_3 represent the acceleration input on each axis, which depends on the applied input voltages, as follows

$$\begin{aligned} u_1 &= \frac{blK_v^2(V_2 - V_4)}{I_{xx}} \\ u_2 &= \frac{blK_v^2(V_3 - V_1)}{I_{yy}} \\ u_3 &= \frac{dK_v^2(V_1 - V_2 + V_3 - V_4)}{I_{zz}} \end{aligned} \quad (24)$$

TABLE I

Quadrotor variables and parameters

Symbol	Meaning	Type	Units
ϕ	Roll angle	Measured	rad
$\dot{\phi}$	Roll angular velocity	Estimated	rad/s
θ	Pitch angle	Measured	rad
$\dot{\theta}$	Pitch angular velocity	Estimated	rad/s
ψ	Yaw angle	Measured	rad
$\dot{\psi}$	Yaw angular velocity	Estimated	rad/s
V_i	Voltage applied to propeller i	Known input	V
K_v	Transformation constant	54.945	rad s/V
J_r	Rotators inertia	$6 \cdot 10^{-5}$	kgm^2
I_{xx}	Inertia X-axis	0.0552	kgm^2
I_{yy}	Inertia Y-axis	0.0552	kgm^2
I_{zz}	Inertia Z-axis	0.1104	kgm^2
b	Thrust coefficient	$3.935139 * 10^{-6}$	N/Volt
d	Drag coefficient	$1.192464 * 10^{-7}$	Nm/Volt
l	Distance from pivot to motor	0.1969	m
m	Mass	2.85	kg
g	Acceleration due to gravity	9.81	m/s^2
T_s	Sampling time	0.005	s

Each propeller actuation is approximately proportional to the corresponding rotor speed (in turn, to the input voltage). Note the intuitively expected actions: to increase u_1 (roll action), increase V_2 and decrease V_4 in the same amount; to increase u_2 (pitch action) increase V_3 and decrease V_1 in the same amount; to increase u_3 (yaw action) increase V_1 and V_3 and decrease V_2 and V_4, all in the same amount. The yaw action u_3 is the only one that will produce a variation in the gyroscopic effect u_g.

The input voltages V_i, $i = 1, 2, 3, 4$, are limited by the drivers, $V_i \in [V_{\min}, V_{\max}]$, with $V_{\min} = -10\,\text{V}$ and $V_{\max} = 10\,\text{V}$.

IV. Experimental results

The proposed adaptive schema is now experimentally tested on two axis of the previously depicted experimental platform.

A PC running Linux-RT, a soft RTOS distributed with a GNU GPLv2 license is provided to implement the control algorithms, on top of an Ubuntu installation. The communications between the PC and the quadrotor platform were made with a PMC I/O target. Linux RT is a Linux O.S. with a patch whose objective is to minimize the amount of kernel code that is nonpreemptible. In this way, faster sampling periods with more reliable real-time guarantees (reduced sampling period jitter) can be implemented. The controller has been implemented in C++, using the newmat matrix library available in Ubuntu repositories. With the above operating system and an Intel I3 processor at 3.3 GHz, the computing power is ample enough to execute the MRAC adaptive control loop with a sampling period of 5 ms.

The model reference adaptive control scheme is applied to the roll and pitch axis, keeping the yaw angle constant. In this way, the MIMO model (22) linearized around the equilibrium point is reduced to two decoupled double integrator

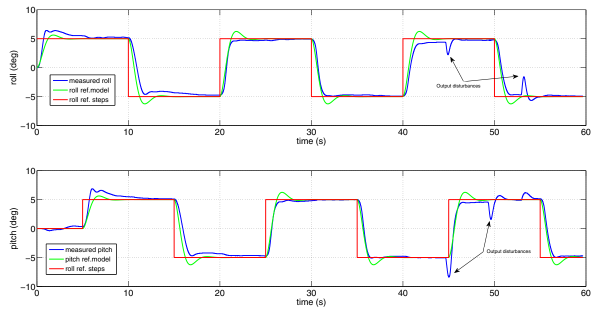

Fig. 10. ML MRAC roll and pitch angles

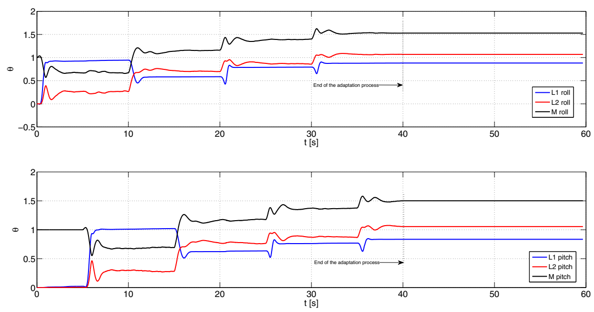

Fig. 11. ML MRAC parameters

subsystems, one for each axis [7]:

$$\ddot{\phi} = u_1$$
$$\ddot{\theta} = u_2$$

The same reference model is used in both axis, which is defined using $\overline{a}_1 = 4.34$, $\overline{a}_2 = 2.3$ and $\overline{b} = 4.34$. Solving $\overline{A}^T P + P\overline{A} = -Q$ for $Q = I_{2\times 2}$ yields to

$$P = \begin{bmatrix} 1.4258 & 0.1152 \\ 0.1152 & 0.2675 \end{bmatrix} \quad (25)$$

This P matrix is then included in the adaptive control law $\dot{K} = -\gamma\Gamma^T Pz$, for Γ given in (15). The experimental response to a square wave reference input u_c and $\gamma = 0.1$ is shown in Fig. 10. The measured output is close to the reference model although they do not match perfectly. However, the system is able to follow the references fairly well. The evolution of the adaptation parameters is shown in Fig. 11. Notice that the parameters convergence is slower than in the simulation examples (here, $\gamma = 0.1$), but an attempt to increase the convergence rate will saturate the quadrotor actuators. The adaptation algorithm was active only during the first two step references. After that moment, the parameters were kept constant. Notice that all the parameters were initialized with a null value (L) and unity (M), i.e., the system starts from an open-loop situation.

V. Conclusions

The MRAC control strategy has been applied for a special kind of systems, those without transmission zeros. This kind of models is frequent in practical applications and it is obtained when linearized electromechanical systems are considered, as it has been illustrated for a lab quadrotor.

An adaptive state feedback control with adjusted input/output gain has been derived, its convergence being proved and experimentally illustrated when applied to the quadrotor.

The adaptation mechanism allows dealing with uncertain plant models and the algorithm provides the estimated controller parameters. They converge to the ideal ones, in simulation. The convergence rate can be tuned by a design parameter (γ), requiring a tradeoff between convergence speed and control action. In practical applications, the control action amplitude is limited and thus, the parameters convergence is slow. If this speed is increased, the actuators saturate and the stability is not guaranteed.

Next step will be to extend the results to a more general class of systems and to investigate the implementation constraints.

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