Attitude Estimation using Low-Cost Sensors: a comparative analysis*

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***Abstract*— In this paper several algorithms to estimate the attitude of a UAS using low-cost sensors are reviewed and a new one including the angular velocity is proposed. The fusion of measurement coming from gyroscopes and accelerometers is based on the Kalman filter. A quadrotor experimental platform is used to compare the measurement systems. The estimation results are evaluated under different conditions and compared against the results obtained with an industrial measurement device.**

***Index Terms*— attitude, estimation, Kalman, sensors, low-cost**

I. INTRODUCTION

In the past years there has been an increasing interest in Unmmaned Aerial Systems (UAS). Among the UAS, quadrotors are of special interest in control from both perspectives, theoretical and applied [1]. They have been used as testbed platforms for validation of non-linear [2], robust and predictive controllers [3]. Disregarding the control strategy, a high-performance attitude tracking subsystem is a requisite for developing any other high-level controlling task. A good example of this statement can be found in [4], where a full control (vision, collision avoidance, landing/taking-off) is developed relying on the attitude control.

The key state variables to be estimated are the attitude and the angular velocity, as they are the primary variables used in attitude control of the vehicle [5]. Inertial Measurement Units (IMUs), which are the core of lightweight robotic applications have experienced a proliferation, resulting in cheaper, and more accurate devices [6]. The emergence of cheaper IMUs makes it possible to use UAS for civil purposes like ground traffic inspection [7], forest fire monitoring [8] or real-time irrigation control [9].

In this paper, low-cost IMUs are considered those devices with a price less than 100 USD. These are very cheap indeed, and they are commonly referred to as hobbyist-level IMUs. These devices have lower performance in terms of bias stability, nonlinearities and signal-to-noise ratio than those on the market for industrial applications. A comparison of a wide range of IMUs can be found in [10].

It is thus both a challenging and interesting task to obtain a reliable attitude estimation using low-cost sensors. This is difficult due to the low performance of the sensors which restricts the quality of the resulting estimation. And it is definitely interesting because the problem of obtaining an accurate attitude estimation is crucial and it usually represents a large portion of the cost of an UAS [6].

The sensor fusion problem consists of obtaining an optimal estimation of the required vehicle state variables with the direct measurements from multiple sensors. There are many possible solutions to this problem, e.g., Kalman filters [11], [12] or complementary filters [13]. This paper aims at providing a comparative evaluation of attitude estimation algorithms using low-cost sensors (gyroscopes and accelerometers). Several aspects must be taken into account while choosing the most suitable approach for a given application: singularity existence, convergence guarantee, computational time, bias estimation, etc. The evaluation will be focused on Kalman filtering methods, as they provide a suitable framework for an easy integration in higher level localization techniques based on laser range finders, cameras or GPS [4].

The major contributions of this paper are to provide a comparative evaluation of different algorithms in the literature and propose a slightly modified algorithm in order to improve the angular velocity estimation. The results show that it is possible to obtain a performance with a hobbyist-grade IMU similar to that of an industrial-grade IMU.

The paper is structured as follows. First, the UAV attitude representation and the measurement problem is reviewed in Section II. Then, the experimental setup is described and different measurement devices are considered in Section III. A review of the most frequently used attitude estimation algorithms is reported in Section IV, introducing a simple modification to also consider the velocity estimation. Finally, Section V summarizes the results after multiple experiments, showing the advantages of the proposed method.

II. BASIS AND NOTATION

A. Problem statement

The problem of attitude estimation consists on recovering the true attitude using the signals provided by the gyroscopes and the accelerometers. The gyroscopes measure the angular velocity of the body, and thus they can be integrated to obtain the attitude. This approach yields to a drift in quite a short time [14], due to the errors introduced by the integration of bias and noise, as it will be discussed below.

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The accelerometers sense the orientation of the gravity acceleration, from which the attitude can be obtained directly. However, the accelerometer signal is highly corrupted with noise due to the vibrations and this approach yields to an estimation too noise to be used in practice. A simple approach that is widely used and provides good results is the complementary filtering, where the accelerometers are low-pass filtered and the gyroscopes are high-pass filtered [15].

In general, filtering algorithms try to fuse the information of both sensors in order to provide a smooth and fast attitude estimation [16]. As it was mentioned before, this paper is focused on Kalman filtering, but different schemes will be tested. Notations and orientation representations are introduced next.

B. Attitude representation

Let us denote by $\{\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3\}$ the unit basis vectors of the Earth-Centered Earth-fixed (ECEF) reference frame, $\{E\}$, which is assumed to be inertial.

Let $\boldsymbol{\omega} = \boldsymbol{\omega}^B_{B/E} = [p, q, r]^T$ be the angular velocity of the aircraft with respect to $\{E\}$ expressed in the body frame $\{B\}$. Thus, the rotational kinematics relating these angular velocities to the Euler angles, $\boldsymbol{\eta} = [\phi\, \theta\, \psi]^T$, is expressed as

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \boldsymbol{\omega} \tag{1}$$

where ϕ, θ, and ψ denote the roll, pitch and yaw angles, respectively.

It is a well-known result that the rate of change of the basis vectors is given by

$$\frac{d\hat{\boldsymbol{e}}_i}{dt} = \boldsymbol{\omega}_{E/B} \times \hat{\boldsymbol{e}}_i = -\boldsymbol{\omega} \times \hat{\boldsymbol{e}}_i = [\boldsymbol{\omega}]^T_\times \hat{\boldsymbol{e}}_i \tag{2}$$

where the skew-symmetric operator is defined as

$$[\boldsymbol{\omega}]_\times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3}$$

According to the roll-pitch-yaw sequence of Euler angles the rotation matrix is expressed as

$${}^{\mathrm{B}}\boldsymbol{R}_E = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - c\phi c\psi & c\phi c\theta \end{bmatrix} \tag{4}$$

where ${}^{\mathrm{B}}\boldsymbol{R}_E$ maps vectors of $\{E\}$ onto $\{B\}$. Noticing that ${}^{\mathrm{B}}\boldsymbol{R}_E = [\hat{\boldsymbol{e}}_1\, \hat{\boldsymbol{e}}_2\, \hat{\boldsymbol{e}}_3]$ and the time derivative of this rotation matrix can be derived by generalizing (2), it follows that the kinematics in terms of the rotation matrix, also referred to as the Direct Cosine Matrix (DCM), is given by

$${}^{\mathrm{B}}\dot{\boldsymbol{R}}_E = [\boldsymbol{\omega}]^T_\times {}^{\mathrm{B}}\boldsymbol{R}_E \tag{5}$$

Rotations are also represented using quaternions, which are an extension to the complex numbers. They are mainly used because they provide a singularity-free representation, and also because quaternion algebra is computationally efficient. The unit quaternion is represented by $\boldsymbol{q} = [\boldsymbol{q_0}\, q_4]^T$ where $\boldsymbol{q_0} = [q_1\, q_2\, q_3]^T$ is the vector part and q_4 is the scalar part of the quaternion. The rigid body angular motion obeys the vector differential equation

$$\frac{d\boldsymbol{q}}{dt} = \boldsymbol{\Omega}(\boldsymbol{\omega})\boldsymbol{q} \tag{6}$$

where

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \frac{1}{2}\begin{bmatrix} [\boldsymbol{\omega}]_\times & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix} \tag{7}$$

The rotation matrix is expressed in terms of the quaternion components as follows

$${}^{\mathrm{B}}\boldsymbol{R}_E = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 - q_3q_4) & 2(q_1q_3 + q_2q_4) \\ 2(q_1q_2 + q_3q_4) & q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 - q_1q_4) \\ 2(q_1q_3 - q_2q_4) & 2(q_2q_3 - q_1q_4) & q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{8}$$

III. Experimental devices

A. Sensor characterization

The output of a MEMS sensor is corrupted by noise and an offset usually referred to as bias [17]. The bias can be calibrated prior to each flight. However, it is dependent on the temperature, and that causes the bias to drift. This effect is specially remarkable within the first few minutes of operation, because of the internal warm-up of the electronic components [18].

For the attitude estimation problem the biases of the gyroscopes are much more crucial than the others. The output of the gyroscopes is time-forward integrated. Even considering an ideal scenario without bias, the integration of the gyro output corrupted by white noise would give rise to an error growing as $\epsilon_w \propto \sqrt{t}$. The addition of an offset term is even worse, originating an error that grows proportionally with time, $\epsilon_b \propto t$. As a consequence, a method is needed to correct the errors introduced by the integration of bias and white noise.

The biases of the accelerometers are not so important because, after a calibration, they will result only in a small offset with respect to the actual horizontal plane, which is perpendicular to the gravity vector.

Let us consider the sensors are modeled as follows,

$$\begin{aligned} \bar{\boldsymbol{\omega}} &= \tilde{\boldsymbol{\omega}} + \boldsymbol{\beta}_\omega + \boldsymbol{\eta}_\omega \\ \bar{\boldsymbol{a}} &= \tilde{\mathbf{a}} + \boldsymbol{\eta}_a \end{aligned} \tag{9}$$

where the velocity measurement $\bar{\boldsymbol{\omega}}$ is composed of its actual value $\tilde{\boldsymbol{\omega}}$, plus the bias $\boldsymbol{\beta}_\omega$ and noise $\boldsymbol{\eta}_\omega$. The same applies for the acceleration measurements but the biases is not included. As mentioned above, this errors are not so critical as they are not integrated over time. The measurement noises are subject to a Gaussian representation as follows,

$$\begin{aligned} \mathbb{E}[\boldsymbol{\eta}_\omega] &= 0 \qquad & \mathbb{E}[\boldsymbol{\eta}_\omega\boldsymbol{\eta}_\omega^T] &= \boldsymbol{\Sigma}_\omega = \sigma_\omega^2\boldsymbol{I}_3 \\ \mathbb{E}[\boldsymbol{\eta}_a] &= 0 \qquad & \mathbb{E}[\boldsymbol{\eta}_a\boldsymbol{\eta}_a^T] &= \boldsymbol{\Sigma}_a = \sigma_a^2\boldsymbol{I}_3 \end{aligned} \tag{10}$$

with $\boldsymbol{\Sigma}_\omega$ and $\boldsymbol{\Sigma}_a$ diagonal covariance matrices.

A random walk process,

$$\dot{\boldsymbol{\beta}}_\omega = \boldsymbol{\eta}_\beta, \tag{11}$$

$$\mathbb{E}[\boldsymbol{\eta}_\beta] = 0, \qquad \mathbb{E}[\boldsymbol{\eta}_\beta\boldsymbol{\eta}_\beta^T] = \boldsymbol{\Sigma}_\beta = \sigma_\beta^2\boldsymbol{I}_3, \tag{12}$$