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Dead-time-compensator for unstable MIMO systems with multiple time delays☆,☆☆

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ABSTRACT

In this paper a new dead-time-compensator to deal with unstable time delay systems is presented. The result is an extension to multiple-input multiple-output systems with multiple and different time delays of a previous result already reported for single-input single-output systems. There are two key issues: the system instability and the presence of different time delays in each signal channel. The proposed approach is developed in three steps. First, a non-delayed output plant is predicted. This predictor is a stable dead-time-compensator coping with multiple and arbitrary delays in all the signal channels. Then, a stabilizer controller is easily designed for the resulting non-delayed plant. For this stabilized plant, the control performance is improved in order to achieve some output tracking and regulation requirements. The results are illustrated by two examples showing their applicability to unstable multiple-input multiple-output multi-delayed plants, which is the main novelty of the proposed approach.

1. Introduction

Most industrial processes are characterized by the presence of time delays. These time delays may appear in the input actions as well as in the measurement paths, but also in the interconnection between internal variables. Delays can be also introduced in the controller itself (computation time of the control algorithm, communication networks, sensors and/or actuators induced delays, etc.). As a result, each signal path between outputs and inputs may show a different delay. That is, there is not a plant delay. In general, the control system performance could be very sensitive to these delays, even more than to other parameters in the model [10,18].

The Smith Predictor (SP) [22], initially proposed for simple single-input/single-output (SISO) systems, is a simple solution to improve the performance of classical controllers. The main advantage of the SP method is that plant time delay is eliminated from the characteristic equation of the closed-loop system. Thus, the control design and analysis problem for processes with delay can be translated into one for processes without delay. This idea has been exploited and there are many extensions improving its performance [15]. But it is well-known that the SP presents several problems if the open-loop plant is unstable. The use of the full model of the plant, linked to the internal instability of the prediction, result in failing to stabilize unstable systems [17].

During the last years, the control community has carried out a great effort against such drawbacks. Different modifications have been proposed [7,8,24], but neither of the proposed schemas eliminate the time delay from all the sensitivity function of the closed-loop system. Recently in [3,14] two new generalized methodologies for SISO time delay systems have been proposed. Both techniques are based on a stable undelayed output prediction in the discrete time framework.

Dealing with multiple-input multiple-output (MIMO) systems is much more involved, if there is not a single common time delay [20], that is, if there are different time delays in different signal channels [4]. Subsystem interactions also make the control design more challenging.

Under some restrictive conditions [19], such as stable processes, some results were reported in [16] as well as the "generalized multi-delay compensator (GMDC)" proposed by Jerome and Ray [5]. In this paper, in order to simplify the design, the model is split into two parts: the so-called fast (non-delayed) part, and the delayed part. This presumes the factorization of the global transfer function. If this is not the case, the "fast" part may include some internal reduced delays. So, the input delays are split into a common part, denoted as synchronous delays [4], expressed as an input diagonal matrix, and the rest, denoted as asynchronous delays, being included into the fast part. But this makes difficult to tune the con-

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troller. The GMDC can also deal with RHP zeroes and with the more general class of models that allow time delays to appear in any form (such as a sum of signals with different delays), but it fails in dealing with unstable systems [5].

In [23] a robust control design procedure is provided. It can be applied to the set of models that can be factorized into a rational MIMO model in series with left/right diagonal (multiple) delay matrices. Additional delays may be introduced in the input channels to allow the transfer function matrix factorization. The robustness perspective in the design allows to consider these additional delays as uncertainties in the delays. Again, the approach is applicable only for open-loop stable plants. Another design alternative is the use of a decoupling technique in order to convert the design problem into a set of SISO control design problems [6,11,16,25,26]. In any case, the main drawback all of these methods is the use of the full model of the plant to predict the non-delayed output. Thus, they cannot be used to control unstable MIMO time delay systems [19].

In [15], a modification on the scheme proposed by [5] to control an integrative process is presented. This schema includes an additional diagonal matrix transfer function filter in both the actual measurement output and the delayed prediction output.

Unlike the SP based control which can only handle stable plants, Model Predictive Control [2] can handle unstable MIMO systems with multiple delays, even taking into account constraints. However, as it is shown in [12,13,15], the robustness of MPC can be improved if a dead-time-compensator (DTC) is used in the predictor structure.

In this paper, a control design procedure to deal with unstable MIMO plants with multiple and different delays in each signal channel is presented. The procedure implies three steps: (1) first, a stable DTC provides a delay-free computed output. (2) Based on this computed output, that is, for a non-delayed input/output model of the plant, any classical MIMO stabilizing control design technique can be applied to get a stable system. (3) In order to cope with the actual output plant, an additional control loop allows to improve tracking and regulating performance on the plant already stabilized by using the computed output. Robustness issues can be tackled by this external loop.

One example illustrates the control design for tracking and regulation of an unstable MIMO multi-delay plant and a classical process already discussed in [15] is used to show the applicability of the proposed methodology for any MIMO system with multiple and different delays.

2. Problem formulation

A MIMO multi-delayed plant can be represented by

$$y(s) = P(s)u(s) \quad (1)$$

where $u \in R^m$ is the input vector, $y \in R^m$ is the output vector and the elements of the transfer function matrix, $p_{ij}(s) = g_{ij}(s)e^{-L_{ij}s}$, show different time delays, L_{ij}. Although most of the development is applicable to square or rectangular transfer matrices, a square one is assumed to deal with the input/output pairing and decoupling.

In [5] the following properties of the Smith Predictor are introduced: (1) the time delay is eliminated from the closed-loop characteristic equation, (2) for set-point changes, it provides the controller with an immediate prediction of the effects of its control action on the system output, which is forecasted L time units into the future: $\bar{y}(t) = y(t + L)$, and (3) the SP implicitly factors the plant into two parts: $p(s) = g(s)e^{-Ls}$, the first is the rational part of the model.

These properties are not always fulfilled for the extension of the SP to MIMO systems. Let us review some of the available solutions for MIMO stable systems.

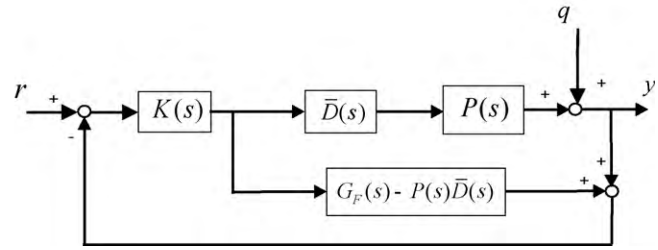

Fig. 1. Generalized multi-delay compensator (GMDC).

For open-loop stable plants, some solutions are proposed under special delay arrangements. If the delays in every row of the transfer function matrix (output delays) are identical, the plant model (1) can be factorized as $P(s) = D_o(s)G(s)$, where $D_o(s) = diag\{e^{-L_{ii}s}\}$ and $G(s)$ is a rational transfer function matrix. The undelayed output is given by $\bar{y}(s) = G(s)u(s)$. Then any typical SP control structure implementing any controller $K(s)$ designed based on the stable fast part model $G(s)$, will solve the problem. Thus, the closed-loop stability is determined by $I + G(s)K(s)$ which contains no delays. In this case the properties above enumerated are fulfilled.

If the delays are the same for each input channel, we still can write $P(s) = G(s)D_i(s)$, but if a controller is designed based on the SP setting, the controlled output does not represent the actual output y at any specific time. It is a totally fictitious value composed of certain "previous output" variables, due to the internal variables coupling. Thus, the system is stabilized but strong interactions and cross delays could lead to unexpected behavior.

In this case, a simple solution is proposed in [5]. It consists in extracting from P a matrix with the shortest dead time in each row appearing in the main diagonal, $D_o(s)$. Then the solution involves describing the fast model D_F (see Fig. 1), including the remaining delays, such that $P = D_oD_F$. In this way, for large controller gain, $|K| \to \infty$, the closed-loop transfer function matrix becomes $H = PK[I + G_FK]^{-1} \to \simeq PK[G_FK]^{-1} = PG_F^{-1} = D_o$. Then the design is as before but G_F includes some delays. The controller K can be a diagonal matrix of PI controllers tuned by using *any* classical or specific procedure proposed in the literature for the design of PID controllers for MIMO plants [15].

The problem is more involved if all the signal channel delays are different. In this case, there is not a unique solution for all cases. A possible solution is to introduce additional delays in the delay matrix $\bar{D}(s)$ leading to the same delay for each input.

The Generalized Predictive Control based on the SP [12], has been shown to be an appropriate tool to deal with these plants, increasing the robustness of the controlled plant but, as pointed out in this reference, it applies for open-loop stable plants.

As previously mentioned, there have been several partially successful attempts to generalize the SP for the control of MIMO systems [23]. In any case, neither of these approaches can cope with the general case of *unstable* MIMO plants with different delays. As for the SISO case, the predictor-scheme should be stable to guarantee the closed-loop stability [19].

In what follows, the case of unstable MIMO plants with different delays in different signal channels is treated. A known model of the plant is assumed in both the rational transfer function and the different delays.

3. DTC for unstable MIMO systems with multiple time delays

Let us first summarize the approach already reported in [3] dealing with unstable SISO delayed system, $p(s) = g(s)e^{-Ls}$, where $g(s)$

is a rational transfer function and L is the time delay. As the information processing is done digitally, a zero-order hold device at the plant input and a sampler with sampling period h at the output will be considered. The internal model of the plant is (A_c, b_c, c), plus the delay.

3.1. Unstable SISO delayed systems

The key feature is to provide an undelayed process output estimation based on input/output measurements filtered through stable filters, regardless the plant poles and zeros location.[1] The discretized model of the delayed SISO system is

$$y(z) = p(z)u(z) \tag{2}$$

$$y(z) = \frac{n(z)}{d(z)} z^{-d} u(z) = g(z) z^{-d} u(z) \tag{3}$$

where $g(z) = c(zI - A)^{-1} b$ is the transfer function[2] based on the internal representation (A,b,c) and $d = L/h$ is the discrete time (DT) delay assumed to be an integer without loss of generality.

The idea, in this section, is to use the undelayed part of the process model, $g(z)$, to define a stable and realizable predictor to compute an output without delay.

Let us define an undelayed system signal such as:

$$y^{\dagger}(z) \doteq g^{\dagger}(z)u(z); \qquad g^{\dagger} = \frac{n^{\dagger}(z)}{n(z)} g(z) = \frac{n^{\dagger}(z)}{d(z)} \tag{4}$$

where $n^{\dagger}(z)$ is going to be determined.

This signal (4) should be computed from the plant input and output data. That is

$$y^{\dagger}(z) = \psi_d(z)u(z) + y(z) \tag{5}$$

The following lemma provides a solution.

Lemma 3.1 ([3]). *A delay-free computed output signal, $y^{\dagger}(z)$, can be obtained by (5) if the filter $\psi_d(z)$ is given by*

$$\psi_d(z) = cA^{-d} \sum_{i=1}^{d} A^{i-1} b z^{-i} \tag{6}$$

Thus, this model based estimator is stable, even for unstable and/or non-minimum phase systems.

Remark. Note that combining (2) and (4) it yields

$$g^{\dagger}(z) = \psi_d(z) + g(z) z^{-d} \tag{7}$$

Thus,

$$\frac{n(z)}{d(z)} z^{-d} = -\psi_d(z) + \frac{n^{\dagger}(z)}{d(z)}$$

That is, the predictor filter $\psi_d(z)$ can be obtained as minus the exact quotient $n(z)z^{-d}/d(z)$, whereas $n^{\dagger}(z)$ expresses the residue.

3.2. Unstable multi-delayed MIMO systems

Let as considered a MIMO system with m-inputs and m-outputs represented by the transfer function matrix

$$P(z) = \begin{bmatrix} p_{11}(z) & \cdots & p_{1m}(z) \\ \vdots & \ddots & \vdots \\ p_{m1}(z) & \cdots & p_{mm}(z) \end{bmatrix} \tag{8}$$

[1] Remind that the main drawback of the SP for open-loop unstable plants is the use of the plant model to generate the undelayed output.

[2] $A = e^{A_c h}$, and $b = \int_0^h e^{A_c \sigma} d\sigma \, b_c$.

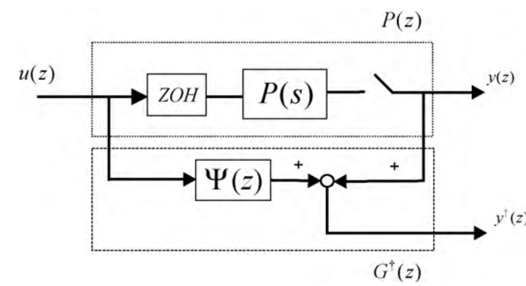

Fig. 2. Delay-free computed output.

whose elements are

$$p_{ij}(z) = g_{ij}(z) z^{-d_{ij}} \tag{9}$$

being $g_{ij}(z)$ the undelayed transfer function corresponding to the output–input pair ij, that is $y_i(z) = \sum_{i=1}^{m} g_{ij} z^{-d_{ij}} u_j(z)$.

Some of these elements are assumed to be unstable. Thus, any estimator based on the use of these elements will be unstable and the corresponding SP will not be internally stable.

Let us denote by matrix $G(z) = [g_{ij}(z)]$ the rational model of the plant, with a minimal internal representation (A,B,C), that is:

$$G(z) = C(zI - A)^{-1} B = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix} (zI - A)^{-1} \begin{bmatrix} b_1 & \cdots & b_m \end{bmatrix} \tag{10}$$

Define:

$$\Psi(z) \doteq \begin{bmatrix} \psi_{11}(z) & \cdots & \psi_{1m}(z) \\ \vdots & \ddots & \vdots \\ \psi_{m1}(z) & \cdots & \psi_{mm}(z) \end{bmatrix} \tag{11}$$

with elements

$$\psi_{ij}(z) = c_i A^{-d_{ij}} \sum_{k=1}^{d_{ij}} A^{k-1} b_j z^{-k} \tag{12}$$

defined as in (6) for the SISO case.

Lemma 3.2. *Given a MIMO* ***unstable*** *plant, (8), with input vector $u(z)$ and output vector $y(z)$, with multiple and different delays in each signal path, d_{ij}, a non-delayed output $y^{\dagger}(z)$ can be computed by means of a stable DTC such that*

$$y^{\dagger}(z) = G^{\dagger}(z)u(z)$$

where the "auxiliary" transfer function matrix

$$G^{\dagger}(z) = \begin{bmatrix} g^{\dagger}_{11}(z) & \cdots & g^{\dagger}_{1m}(z) \\ \vdots & \ddots & \vdots \\ g^{\dagger}_{m1}(z) & \cdots & g^{\dagger}_{mm}(z) \end{bmatrix} \equiv (\Psi(z) + P(z)) \tag{13}$$

with elements $g^{\dagger}_{ij}(z) \equiv \psi_{ij}(z) + p_{ij}(z)$.

Proof. Applying Lemma 3.1, to each element (9) in (8), a non-delayed computed plant output, $y^{\dagger}(z) = G^{\dagger}(z)u(z)$, is obtained, as illustrated in Fig. 2. □

As suggested in the Remark following Lemma 3.1, each element $\psi_{ij}(z)$ can be computed as the quotient of the corresponding polynomials.

Remark. The auxiliary transfer function matrix, $G^{\dagger}(z)$, provides an output without delays with respect to the inputs. Thus, as later seen, it paves the way to stabilize the system. The price we pay for that is complex filtering ($\Psi(z)$) and high gains. If some delays are left in $G^{\dagger}(z) \rightarrow G^{\dagger}_F(z)$, as proposed in [5], some simplifications

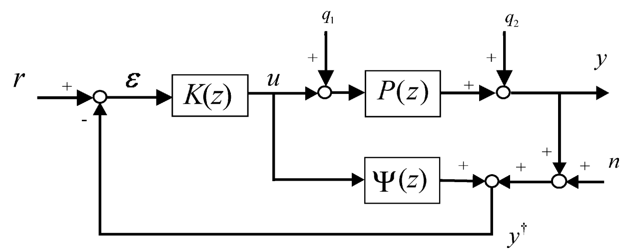

Fig. 3. Unstable MIMO stabilization.

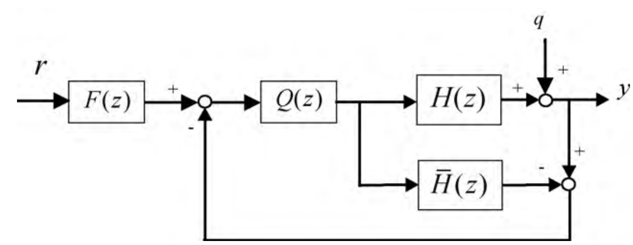

Fig. 4. Proposed DTC-IMC for the control of stabilized delayed MIMO systems.

can be obtained. First some delays can be homogenized, leading to common input or output delays. Also, the filter order is reduced. Again, the price we pay for that is a more difficult stabilizer design.

Even this output is not the actual plant output, for the sake of stabilization, an appropriate controller can be designed without considering the multiple and different delays.

4. Stabilization of unstable MIMO systems with multiple time delays

Now we are in a position to stabilize MIMO systems with multiple and different delays in the signal channels. Based on the delay-free computed output, $y^{\dagger}(z)$, any classical control design technique can be used to stabilize the plant.

From Fig. 3, the following output/reference response is obtained:

$$y(z) = P(z)K(z)[I + (\Psi(z) + P(z))K(z)]^{-1}r(z) \tag{14}$$

and taking into account (13) it yields:

$$y(z) = P(z)K(z)[I + G^{\dagger}(z)K(z)]^{-1}r(z) \tag{15}$$

Denoting the sensitivity function as

$$S_m(z) = [I + G^{\dagger}(z)K(z)]^{-1} \tag{16}$$

the controlled plant transfer function matrix is

$$y(z) = P(z)K(z)S_m(z)r(z) = H(z)r(z) \tag{17}$$

Remark. In order to stabilize the plant $P(z)$, the controller $K(z)$ in Fig. 3 is designed such that the matrix

$$[I + G^{\dagger}(z)K(z)] \tag{18}$$

is Schur.[3]

Although the plant is unstable, the controlled plant depicted in this figure is internally stable. In fact, all the partial transfer matrices T_{yr}, T_{yq_1}, T_{yq_2}, T_{yn}, T_{ur}, T_{uq_1}, T_{uq_2} and T_{un} are stable.

Lemma 4.1 ((Internal stability)). *Given the plant (8), any controller $K(z)$ implemented as in Fig. 3, designed to stabilize the undelayed plant model (13), that is, such that (18) is Schur, provides internal stability to the controlled plant.*

Proof. From Fig. 3 and the equivalence (13) (Lemma 3.2), the following expressions are obtained:

$$\varepsilon = S_m[r - q_2 - Pq_1 - n] \tag{19}$$

$$u = KS_m[r - q_2 - Pq_1 - n] \tag{20}$$

$$y = H[r - n] + [I - H]q_2 + [I - H]Pq_1 \Rightarrow y = H[r - n] + [I + \Psi(z)]S_m q_2 + [I + \Psi(z)]S_m Pq_1 \tag{21}$$

Thus, as P and $G^{\dagger}$ (in S_m) have the same poles, stability is guaranteed as far as K is designed to make H stable (17). Any classical control design approach can be used. □

At this stage, the initial problem has been translated into the control of the multi-delayed stable MIMO system H, and some of the design approaches discussed in Section 2 can be applied. So far, the problem of plant interactions has not been solved and even the steady-state behavior has not been considered.

As already pointed out, although the computed non-delayed output $y^{\dagger}$ can be controlled by K to fulfil some requirements concerning the system behavior, the actual output is a mixed combination of partial delayed outputs and the final result could be rather unsatisfactory. It is worth to note that the stabilized transfer function matrix, $H(s)$, can be split into

$$H(z) = P(z)M(z) \tag{22}$$

$$M(z) = K(z)[I + G^{\dagger}(z)K(z)]^{-1} \tag{23}$$

By design, H is stable but the product $P.M$ will introduce additional delay interactions. That is, if M is not diagonal, the elements of matrix H will show terms with different delays. This issue has been treated in [5], but obviously it leads to some undesirable complexity in the global control design.

5. Control of unstable MIMO systems with multiple time delays

Let us consider some other features of the controlled plant.

5.1. Steady-state behavior

First, by designing $K(z)$, the plant has been stabilized, but the static DC gain of the controlled plant cannot be robustly tuned.

In fact, for unitary reference ($r = (z/(z-1))I$) the steady-state output is (17),

$$\lim_{k\to\infty} y_k = H(1) \tag{24}$$

and, even for infinity gain controller, $H(1) \neq \mathbf{I}$. Thus, a steady-state error appears. The same happens for constant disturbances.

As the controlled plant, H, is internally stable an extra controller can be designed, as for example by the IMC design technique, to improve tracking and disturbance rejection robustness.

Let us consider a generic output disturbance q. A possible control structure is depicted in Fig. 4, where $F(z)$ is a reference pre-filter to smooth the input. The plant output is obtained from

$$y = HQ(Fr - q) + q \tag{25}$$

where Q and H are stable transfer function matrices (in the ideal case, it is assumed that the stabilized plant model matches the designed one, $\bar{H} = H$). Following [9] the system is internally stable if H and Q are stable.

In this case, the error can be computed from (25) as

$$e = y - Fr = \{HQ - I\}(q - Fr) \tag{26}$$

[3] For simplicity in the notation, in the following, the argument of the functions (z) is suppressed, if there is no ambiguity.

Thus, in order to cancel the steady-state error it suffices $Q(1) = [H(1)]^{-1}$.

5.2. Robust stability

If the delay is dominant in the dynamic behavior [1], the auxiliary transfer function matrix $G^{\dagger} \equiv P(z) + \Psi(z)$, may involve high gains, reinforcing the coupling between variables, even if they were not so interactive in the original plant, $P(z)$.

In these cases, it would be interesting to combine the proposed approach with some others where extra delays are incorporated in the fast transfer function matrix, like the already mentioned GMDC. This idea has been also used to control MIMO systems with integrators [15].

Note that the design of the external loop controller Q, can be focussed into reducing the coupling and getting some extra required *robust* performance.

Combining (17) and (26), it yields

$$e = [PKS_mQ - I](q - Fr) \tag{27}$$

Assuming multiplicative output uncertainties, the process can be modeled by $P_p(z) = (1 + W_o(z))P(z)$, where $W_o(z)$ is assumed to be stable. The robust stability condition can be expressed by[4]

$$||PKS_mQ||_\infty < \frac{1}{||W_o||_\infty} \tag{28}$$

This suggest to split the Q matrix into two terms:

$$Q = Q_0F_0 \tag{29}$$

such that Q_0 is a real matrix to fulfill the requirement $Q_0(1) = [H(1)]^{-1}$, and F_0 is a filtering matrix with unitary static gain, to re-shape the sensitivity matrix. In general, F_0 is defined as

$$F_0 = diag\{f_{11}, \ldots, f_{nn}\}.$$

This filter could be also put in the feedback loop (see [15]).

5.3. Design procedure

In summary, the proposed control design methodology involves:

- Given the DT full plant model (8) and following the sequence in Section 3.2,
  - Compute the rational transfer function matrix $G(z)$,
  - Compute the predictor filter $\Psi(z)$ (11),
  - Get the auxiliary transfer function matrix $G^{\dagger}(z)$ (13).
- Stabilize the MIMO system by designing the controller $K(z)$, for $G^{\dagger}(z)$ (13), such that the controlled plant $H(s)$ is stable (17).
  This design is delay-free and *any* suitable control design technique can be applied. Additional requirements on K could lead to a diagonal dominant matrix M (23).
- For the previously stabilized plant, $H(s)$, improve the controlled system performance by, for instance, designing a robust performance controller $Q(z)$ (29).

6. Examples

By using the proposed approach, unstable multi-delayed MIMO systems can be controlled, regardless the position of the unstable element in the transfer function matrix. This is illustrated in the first example. Moreover, the proposed methodology is applied to the example treated in [15], with the same data and conditions used there, to show its applicability to any MIMO multi-delayed unstable system.

Example 1. Let us first consider an unstable MIMO plant. Assume the following plant model, without uncertainties

$$P(s) = \begin{bmatrix} \dfrac{e^{-0.5s}}{s-1} & \dfrac{0.5e^{-0.7s}}{s+1} \\ \dfrac{0.1e^{-0.3s}}{10s+1} & \dfrac{e^{-0.7s}}{s} \end{bmatrix}$$

A DT plant model, assuming a sampling period $h = 0.1s$, (8) is

$$P(z) = \begin{bmatrix} \dfrac{0.10517z^{-5}}{z-1.105} & \dfrac{0.04758z^{-7}}{z-0.9048} \\ \dfrac{0.000995z^{-3}}{z-0.99} & \dfrac{0.1}{z-1} \end{bmatrix}$$

The undelayed output generator (13) is given by

$$G^{\dagger}(z) = \begin{bmatrix} \dfrac{0.063786}{z-1.105} & \dfrac{0.095829}{z-0.9048} \\ \dfrac{0.0010249}{z-0.99} & \dfrac{0.1}{z-1} \end{bmatrix}$$

Let us design a stabilizing H_∞ controller K for the plant $G^{\dagger}$ to shape the sigma plot of the loop transfer function $G^{\dagger}K$ to have the desired loop shape[5]

$$G_d(z) = \begin{bmatrix} \dfrac{0.1}{z-1} & 0 \\ 0 & \dfrac{0.1}{z-1} \end{bmatrix}$$

The resulting controller is a high order controller but it could be simplified to a set of P/PID controllers.[6] As a result, the following controller is chosen

$$K(z) = \begin{bmatrix} \dfrac{3.8625(z-0.9823)}{z-1} & 0.97215 \\ -0.070359 & 0.58583 \end{bmatrix}$$

leading to $H(z) = P(z)K(z)[I + G^{\dagger}(z)K(z)]^{-1}$, which is stable. The controlled system static gain is:

$$H(1) = \begin{bmatrix} 1.6487 & 0 \\ 0.0050 & 1.0002 \end{bmatrix}$$

Thus, the IMC controller is chosen such that $Q = Q_0$ with

$$Q_0 = H(1)^{-1} = \begin{bmatrix} 0.6065 & 0 \\ -0.0030 & 0.998 \end{bmatrix}$$

The reference step responses are plotted in Fig. 5. A pre-filter $F(z)$ to smooth the set-point tracking have been included in the reference signals,

$$F(z) = \begin{bmatrix} \dfrac{0.0198}{z-0.9802} & 0 \\ 0 & \dfrac{0.0198}{z-0.9802} \end{bmatrix}$$

Now, in order to illustrate the possibility of robustness improvement by using the Q-controller, let us consider a disturbed plant

[4] Note, that if another kind of uncertainties is considered, a similar robust condition index is obtained [21].

[5] The Matlab® command loopsyn allows this design.

[6] By model reduction techniques, as implemented for instance in Matlab®, getting a minimal realization (minreal) and then reducing the order to one (balreal).

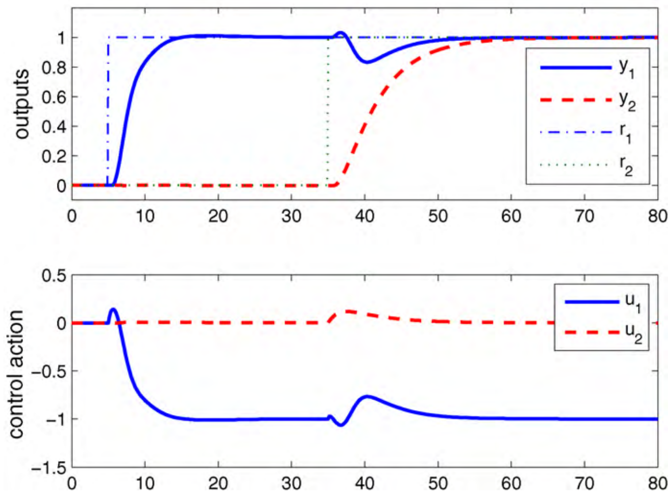

Fig. 5. Step responses Example 1. $K(z)$ being designed by loop shaping.

such that the real process is:

$$P_p(s) = \begin{bmatrix} \dfrac{e^{-0.6s}}{s-1.1} & \dfrac{0.6e^{-0.8s}}{s+1.1} \\ \dfrac{0.1e^{-0.3s}}{10s+1} & \dfrac{1.1e^{-0.7s}}{s} \end{bmatrix}.$$

If the previous controllers (K and Q) are used, the system becomes unstable, but if the Q-controller is chosen such that $Q = Q_0 F_0$ with

$$F_0(z) = \begin{bmatrix} \dfrac{0.01}{z-0.99} & 0 \\ 0 & \dfrac{0.01}{z-0.99} \end{bmatrix},$$

the result in Fig. 6 are obtained. Note that if the filter is included, the close loop is stable being the set-point response almost the same as before. Thus, the new controller improves the robustness performance.

The formal design of this controller is a matter of current research.

It is worth to note in the figure that there are some loop interactions. This issue is also a matter of further research. In any case, if an improvement in the decoupling response is required, it is possible to combine the proposed scheme with the design procedure proposed in [5].

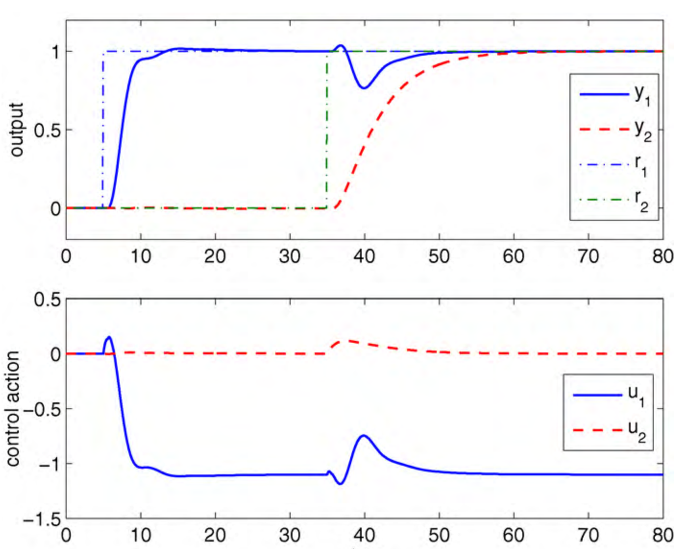

Fig. 6. Step responses for parametrical uncertainties.

Example 2. Let us considered the level and temperature control in a three-stage evaporator system already studied in [15].

The process model is:

$$P(s) = \begin{bmatrix} \dfrac{3.5e^{-1s}}{s} & \dfrac{-1e^{-5s}}{2s+1} \\ \dfrac{2^{-7s}}{1.5s+1} & \dfrac{-1e^{-5s}}{3.2s+1} \end{bmatrix}, \qquad P_q(s) = \begin{bmatrix} \dfrac{3.5}{s}e^{-3s} \\ \dfrac{-4.5}{2s+1}e^{-2s} \end{bmatrix}$$

$$y(s) = P(s)u(s) + P_q(s)q(s)$$

where the outputs are the level and temperature in the final storage tank, and the inputs are the juice and steam flows. The input disturbance $q(s)$ is the output flow in the tank. The free-delay model is:

$$G(s) = \begin{bmatrix} \dfrac{3.5}{s} & \dfrac{-1}{2s+1} \\ \dfrac{2}{1.5s+1} & \dfrac{-1}{3.2s+1} \end{bmatrix}$$

whereas the proposed undelayed output $y^\dagger$ is obtained by (13):

$$G^\dagger(s) = \begin{bmatrix} \dfrac{3.5}{s} & \dfrac{-24.3640}{2s+1} \\ \dfrac{-212.7}{1.5s+1} & \dfrac{-4.7712}{3.2s+1} \end{bmatrix}$$

Note that higher gains appear in $G^\dagger$ due to the large delays. The good news are that this matrix can be used even for unstable plants. To reduce these gains, some delays can be taken out of the predictor, as suggested in the GMDC.

For instance, we could assume the following fast model[7]

$$G_F^\dagger(s) = \begin{bmatrix} \dfrac{3.5}{s} & \dfrac{-2.6380e^{-4s}}{2s+1} \\ \dfrac{7.5869^{-5s}}{1.5s+1} & \dfrac{-4.7}{3.2s+1} \end{bmatrix} \tag{30}$$

In [15], the following fast model is considered

$$G_F(s) = \begin{bmatrix} \dfrac{3.5}{s} & \dfrac{-1e^{-4s}}{2s+1} \\ \dfrac{2^{-2s}}{1.5s+1} & \dfrac{-1}{3.2s+1} \end{bmatrix} \tag{31}$$

The scheme in Fig. 4 is implemented, with sampling period $T = 0.1$ s, where the predictor filter $\Psi(z)$ is computed as

$$\Psi = \begin{bmatrix} \psi_{11}(z) & \psi_{12}(z)z^{-4/T} \\ \psi_{21}(z)z^{-5/T} & \psi_{22}(z) \end{bmatrix}$$

with $\Psi_{ij}(z)$, (6), being computed from the discretized delay-free model of (30). Note that

$$\Psi \equiv \begin{bmatrix} g_{11}^\dagger(z) - p_{11}(z) & (g_{21}^\dagger(z) - p_{12}^\dagger(z))z^{-4/T} \\ (g_{21}^\dagger(z) - p_{21}^\dagger(z))z^{-5/T} & g_{12}^\dagger(z) - p_{12}(z) \end{bmatrix}$$

The following Q-controller is applied

$$Q = H(1)^{-1} = \begin{bmatrix} 1 & 0.6533 \\ 0 & 4.8216 \end{bmatrix}$$

[7] For the sake of comparison, G, G_F, $G^\dagger$ and $G_F^\dagger$ are expressed as a function of s.

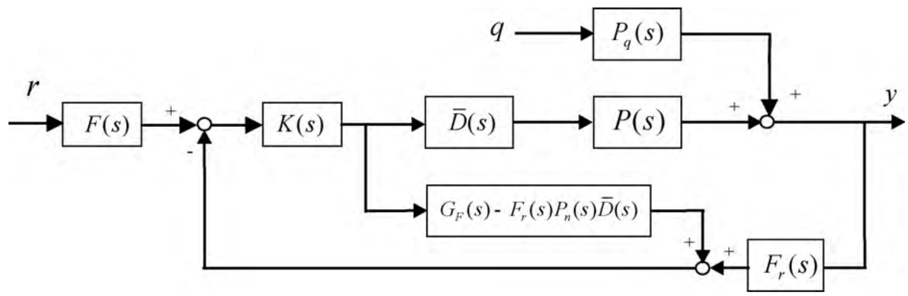

Fig. 7. Filtered GMDC [15].

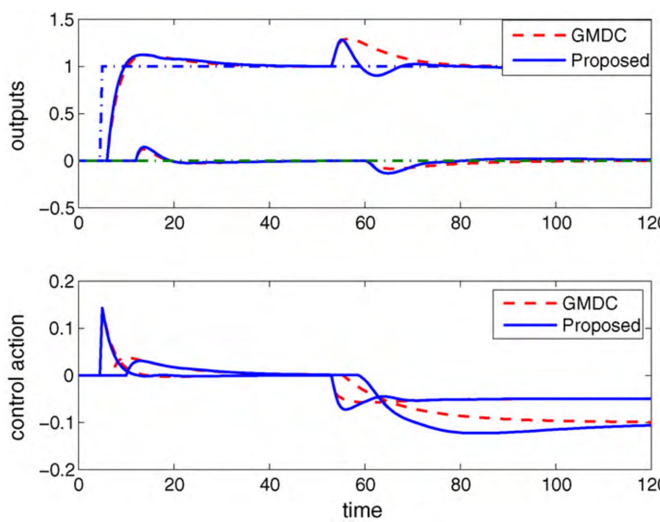

Fig. 8. Comparison between the GMDC proposed in [15] and the DTC-based proposed here. There is an initial set-point change in y_1 at $t = 5$ s, as well as a disturbance in $q = 0.5$, at $t = 50$.

The proposed scheme is compared with the one proposed by [15] (see Fig. 7), where $\bar{D} = \mathbf{I}$, the fast model is (31) and diagonal matrix:

$$F_r(s) = \begin{bmatrix} \frac{1.4s+1}{(0.2s+1)^2} & 0 \\ 0 & 1 \end{bmatrix}$$

The same controller design technique suggested in [15] is used to compared both control schemes, that is,

$$K(s) = \begin{bmatrix} \frac{k_{11}(T_{11}s+1)}{T_{11}s} & 0 \\ 0 & \frac{k_{22}(T_{22}s+1)}{T_{22}s} \end{bmatrix}$$

the parameters are $k_{11} = 0.5/3.5$, $T_{11} = 10$, $k_{22} = -5/4.7$, $T_{22} = 3.2$, and $k_{11} = 0.5/3.5$, $T_{11} = 10$, $k_{22} = -5$, $T_{22} = 3.5$ in [15]. k_{ij} and T_{ij}, are the parameter of $g_{ij} = k_{ij}/(T_{ij}s+1)$, in (30) and (31) respectively.

The obtained results are shown in Fig. 8. Note that in both schemas, to achieve some required performance, the K-parameters can be easily retuned.

7. Conclusions

The presence of multiple and different delays in the input/output signal paths makes the stabilization of unstable MIMO plants a difficult task. The use of dead-time-compensator based on the SP setting fails due to the plant unstability.

In this paper, a MIMO DTC suitable for any linear plant has been presented. It is applicable for stable and unstable plants, turning the problem of controlling a MIMO multi-delay unstable plant to one for stable plants. Then, some already available control design approaches can be applied to the stabilized plant.

Interaction between variables can be treated by means of ad hoc approaches and it is a matter of further research.

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