# Robustness of a discrete-time predictor-based controller for time-varying measurement delay \*

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**Abstract:** Robustness properties for different uncertainties of a predictor-based control of time-delay systems are analyzed in this paper. First, a time-varying delay dependent stability condition is expressed in terms of LMIs. Then, uncertainties in the knowledge of the plant parameters and the sampling time period are considered. In addition, the resulting closed-loop system is shown to be robust with respect to these uncertainties. Moreover, this scheme has been tested in a real-time application to control the roll angle of a quad-rotor mini-helicopter. The experimental results have demonstrated the good performance of the proposed scheme and the robustness even in presence of long delays uncertainties.

*Keywords:* Unstable time-delay systems; Time-varying delay-dependent stability; Jitter in digital implementation; Linear matrix inequality (LMI).

### 1. INTRODUCTION

In designing any control system the process behavior imposes some unavoidable performance limitations (see, for instance (Seron *et al.* (1997))). This is clearly the case when dealing with systems with time delays. The Smith Predictor (SP) (Smith (1959)) and the Finite Spectrum Assignment (FSA) (Manitius & Olbrot (1979)) may be considered as the main control methods for linear processes with time delay in either the input or the output, see (Gu & Niculescu (2003); Richard (2003)). A careful analysis of these methods and their modifications show that they all use, in an explicit or explicit manner, prediction of the state in order to achieve the control of the system.

As explained in (Palmor (1996)), the use of an explicit unstable prediction model in the SP approach determines the internal stability of the closed-loop system. Palmor also suggested how to implement the control law using the so-called integral form. In (Manitius & Olbrot (1979)), this approach was also introduced in the framework of spectrum assignment with distributed delays. However, as shown in (Mondie *et al.* (2001)), the implementation of these control approach on a digital computer can result in an unstable behavior. In the survey paper presented by (Richard (2003)), this problem was considered as one of the open problems in the control of time-delay systems. Afterwards, in (Zhong (2004)), two approaches are introduced to approximate the distributed delay, one in the *s*-domain and another in the *z*-domain. Additionally, in (Gudin (2007)), it is shown that, the initial reported difficulties to implement distributed delays using a digital hardware where caused by the incorrect approximation methods. This fact was illustrated experimentally using a pendulum and the approximated control law proposed by (Mirkin (2004)). In this work it is also analyze another approximation in the *s*-domain proposed by (Mondie *et al.* (2001b)).

In (Lozano et al. (2004)), a discrete-time (DT) controller for continuous-time (CT) plants with time delay is proposed and the closed-loop stability is proved. The robustness with respect to small variations in the sampling period, the delay and the delay/sampling period ratio is also proved. Since a computer is normally used to implement the control law, (Astrom & Wittenmark (1997)), it is justified to study whether instabilities may appear in DT control algorithms. Note that small variations in the sampling period may be such that the closed-loop behavior will be described by a quasi-polynomial in the complex variable z. The zero location of quasi-polynomials are known to be very sensitive to small changes in the polynomial parameters and can easily move from the stable region to the unstable region. Therefore it is important to prove robustness also with respect to small variations of the sampling period. The proof of robust stability in the approach presented in (Lozano et al. (2004)) allows

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positive or negative variations in the delay but they must be bounded by the sampling period size.

In (Garcia et al. (2006)) the previous results were enlarged for a general case, i.e. when the delay uncertainties are longer than the sampling period. In this paper, we extend this previous analysis to processes with time-varying delays in the measurement. The real-time performance of the control scheme is illustrated by controlling a lab helicopter prototype.

In this paper, the robustness properties for different uncertainties of a predictor-based control of unstable time-delay systems are analyzed. The problem statement is outlined in the next section. Then, the stability condition for time varying delays is derived and a design approach is proposed. The robustness of the design is then studied. The promising results are applied to a lab helicopter prototype and some experimental results are reported. In the last section, some conclusions are drafted.

#### 2. PROBLEM FORMULATION

Let us consider the following CT state space representation of a delayed system

$$\dot{x}(t) = A_c x(t) + B_c u(t - \tau_1)$$
(1)

$$y(t) = Cx(t - \tau_2(t)) \tag{2}$$

where the nominal plant parameter matrices are  $A_c \in \Re^{n \times n}$ ,  $B_c \in \Re^{n \times m}$ ,  $C \in \Re^{p \times n}$ .  $\tau_1 > 0$  is the control timedelay, assumed to be time-invariant, and  $\tau_2(t) > 0$  is the measurement delay, assumed to be time-varying.

#### 2.1 Discrete time model based predictor

It is assumed that the input/output delay is the same for all the input/output channels. In a computer-based controller implementation the sampling time,  $t_k$ , is periodic, being  $T = t_{k+1} - t_k$  the sampling period.

Discretized variables are denoted as, for instance,  $x_k = x(kT)$  and, without loss of generality, the time delays satisfy

$$\tau_1 = d_1 T, \ (d_1 \in \mathcal{Z}^+) \tag{3}$$

$$\tau_{2_k} = d_{2_k} T, \quad (d_{2_k} \in \mathcal{Z}^+) \tag{4}$$

The delay between the state measurement and the control calculation  $\tau_{2_k} > 0$  is assumed to be randomly time-variant between known lower and upper bounds  $(d_{2_m} \text{ and } d_{2_M}, \text{ respectively})$ , that is,  $d_{2_m} \leq d_{2_k} \leq d_{2_M}$ 

Then, the DT version of (1), is given by

$$x_{k+1} = Ax_k + Bu_{k-d_1}; \quad y_k = x_{k-d_{2_k}} \tag{5}$$

where  $A = e^{A_c T}$ , and  $B = \int_0^T e^{A_c \sigma} d\sigma B_c$ .

For the sake of simplifying the notation, on the sequel we define the entire delay as  $d_k = d_1 + d_{2_k}$  and its lower and upper bounds as  $d_m = d_1 + d_{2_m}$  and  $d_M = d_1 + d_{2_M}$ , respectively.

The control structure proposed here consists of a state feedback law based on the h-step ahead prediction of the

nominal model. Thus, this h ahead state prediction has the following form

$$\bar{x}_{k+h} = A^h x_{k-d_{2_k}} + A^{h-1} B u_{k-h} + \dots + B u_{k-1}$$
 (6)

where  $h \in \mathbb{Z}^+$  is the assumed discrete delay (for example, the mean of  $d_k$ ). Hence, the control law is

$$u_k = -K\bar{x}_{k+h} \tag{7}$$

where  $K \in \Re^{m \times n}$ .

The goal is either to stabilize the closed-loop system (5)-(7) or to improve its dynamic behavior. One of the design parameters is the value of h, that should be in principle as near as possible to the mean of  $d_k$ .

Assumption A1 The lower and upper bounds  $d_m$  and  $d_M$  of the time delay  $d_k$  are perfectly known.

Assumption A2 There are no uncertainties in the process model  $(A_c, B_c)$ .

Assumption A3 The sampling/updating time instants,  $t_k$ , are regular. It is assumed that there is no jitter in the control computation, leading to slightly different sampling/updating periods  $(T_k)$ .

 $Remark \ 1.$  Assumptions A2 and A3 will be treated in Section 4

#### 2.2 Closed-loop time delay system stability

Concerning the stability of the closed-loop system, the following Lemma can be stated.

Lemma 1. The closed-loop system composed of (5), (6) and (7), leads to

$$x_{k+1} = (A - BK)x_k - BKA^h x_{k-d_k} + BKA^h x_{k-h}$$
(8)

where  $\{h, d_k\} \in \mathcal{Z}^+$ . and  $d_m \leq d_k \leq d_M$ 

*Proof 1.* Delaying the control law  $d_1$ -sampling periods, (7) yields,

$$u_{k-d_1} = -K\bar{x}_{k+h-d_1} \tag{9}$$

and by using (6), it yields

$$u_{k-d_1} = -K(A^h x_{k-d_k} + A^{h-1} B u_{k-h-d_1} + \dots + B u_{k-1-d_1})$$
(10)

Introducing (10) into (5) the closed loop realization is obtained as follows

$$x_{k+1} = Ax_k - BKA^h x_{k-d_k} - BK\Phi_k \tag{11}$$

where 
$$\Phi_k = A^{h-1} B u_{k-h-d_1} + \dots + B u_{k-1-d_1}$$
.

On the other hand, the process state,  $x_k$ , depending on  $x_{k-h}$  can be expressed as

$$c_k = A^h x_{k-h} - \Theta_k \tag{12}$$

where  $\Theta_k = A^{h-1}Bu_{k-h-d_1} + \dots + Bu_{k-1-d_1}$ .

Let us observe that,  $\Theta_k$  and  $\Phi_k$  are identical. It is due to the fact that  $d_1$  is time-invariant. Therefore, from (12) and (11) the closed-loop realization can be easily obtained by eliminating the common term  $\Theta_k = \Phi_k$ , leading to

$$x_{k+1} = (A - BK)x_k - BKA^h x_{k-d_k} + BKA^h x_{k-h}$$
(13)

#### 3. TIME-VARYING DELAY-DEPENDENT STABILITY CONDITION

In this section, a robust delay-dependent stability condition using the h-step ahead prediction-based control (7) is proposed. The scheme prediction stabilizes the system subject to time-invariant delays, between controller and process, and also time-variant delays, between the plant and control algorithm, even in presence of time delay uncertainties.

The goal of this section is to propose an analysis procedure (based on the solution of a set of LMIs) to obtain, with low computational  $\cos t^1$ , one sufficient condition to assure the stability of the closed-loop system under bounded variations of the measurement delay.

Define M = (A - BK) and  $A_1 = BKA^h$ , hence, from (8) it yields

$$x_{k+1} = Mx_k + A_1 x_{k-h} - A_1 x_{k-d_k} \tag{14}$$

Theorem 1. For a given control gain, K, designed to stabilize (5)-(7) for some h in the h-step ahead predictor, if there exist positive definite matrices P,  $Q_1$ ,  $Q_2$ ,  $Z_1$  and  $Z_2$ , and matrices  $X_1$ ,  $X_2$ ,  $Y_1$  and  $Y_2$ , such that the following LMI constraints hold

$$\begin{pmatrix} \Gamma & -Y_1 & -Y_2 & M^T P & d_M (M-I)^T Z_1 & h(M-I)^T Z_2 \\ * & -Q_1 & 0 & A_1^T P & d_M A_1^T Z_1 & hA_1^T Z_2 \\ * & * & -Q_2 & -A_1^T P & -d_M A_1^T Z_1 & -hA_1^T Z_2 \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -d_M Z_1 & 0 \\ * & * & * & * & * & -hZ_2 \end{pmatrix} < 0$$

with  $\Gamma:=-P+d_MX_1+hX_2+Y_1+Y_1^T+Y_2+Y_2^T+(d_M-d_m+1)Q_1+Q_2$ 

$$\begin{pmatrix} X_1 & Y_1 \\ Y_1^T & Z_1 \end{pmatrix} \ge 0 \qquad \begin{pmatrix} X_2 & Y_2 \\ Y_2^T & Z_2 \end{pmatrix} \ge 0, \tag{15}$$

then, the close-loop system (14) is asymptotically stable for any time-variant delay,  $d_k$ , satisfying  $d_m \leq d_k \leq d_M$ . *Proof 2.* Denote

$$\nu_j = x_j - x_{j-1}$$

then, the delayed values of the state can be expressed by

$$x_{k-h} = x_k - \sum_{j=k-h+1}^k \nu_j$$

Consequently, (14) can be written as

$$x_{k+1} = Mx_k - A_1 \sum_{j=k-h+1}^k \nu_j + A_1 \sum_{j=k-d_k+1}^k \nu_j \quad (16)$$

Following a procedure similar to that in (Garcia *et al.* (2006)-Gao *et al.* (2008)), let us define the following Lyapunov function candidate,  $V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k)$ , where

$$V_{1}(k) = x^{T}(k)Px(k)$$

$$V_{2}(k) = \sum_{i=k-d_{k}}^{k-1} x^{T}(i)Q_{1}x(k) + \sum_{i=k-h}^{k-1} x^{T}(i)Q_{2}x(k)$$

$$V_{3}(k) = \sum_{j=-d_{M}+2}^{-d_{m}+1} \sum_{i=k+j-1}^{k-1} x^{T}(i)Q_{1}x(i)$$

$$V_{4}(k) = \sum_{i=-d_{M}}^{-1} \sum_{m=k+i}^{k-1} \nu^{T}(m)Z_{1}\nu(m) + \sum_{i=-h}^{-1} \sum_{m=k+i}^{k-1} \nu^{T}(m)Z_{2}\nu(m)$$

Note that an extra term has been included in  $V_2$  and  $V_4$  to take into account both delays, h and  $d_k$ . Then, the system (14) or (8), will be asymptotically stable if

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) < 0$$

After some tedious algebraic manipulation the following result is obtained

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 \le \lambda^T(k)\Omega\lambda(k) \quad (17)$$

where

$$\Omega = \begin{pmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ * & \Psi_{22} & \Psi_{23} \\ * & * & \Psi_{33} \end{pmatrix}$$
(18)

$$\begin{split} \Psi_{11} &= M^T P M + \Gamma + d_M (M-I)^T Z_1 (M-I) \\ &+ h (M-I)^T Z_2 (M-I) \\ \Psi_{12} &= -Y_1 - d_M (M-I)^T Z_1 A_1 - M^T P A_1 \\ \Psi_{13} &= -Y_2 + h (M-I)^T Z_2 A_1 + M^T P A_1 \\ \Psi_{22} &= A_1^T P A_1 - Q_1 d_M A_1^T Z_1 A_1 + h A_1^T Z_2 A_1 \\ \Psi_{23} &= -A_1^T P A_1 - d_M A_1^T Z_1 A_1 - h A_1^T Z_2 A_1 \\ \Psi_{33} &= -A_1^T P A_1 - Q_2 + d_M A_1^T Z_1 A_1 + h A_1^T Z_2 A_1 \end{split}$$

and

$$\lambda^T(k) = (x(k) \ x(k-d_k) \ x(k-h)) \, .$$

After some manipulations and making use of the Schur complement (Boyd *et al.* (1994)), the inequalities (15) are obtained.

*Example 1.* Let us considered the following second order DT system, already studied in (Gao *et al.* (2008)),

$$x_{k+1} = \begin{pmatrix} 0.5450 & -0.0392\\ 0.6225 & -0.1696 \end{pmatrix} x_k + \begin{pmatrix} 1.0000\\ -2.2159 \end{pmatrix} u_{k-d_1}$$

where  $x_k = [x_{1_k} \ x_{2_k}]'$  is the state vector, fully accessible (C = I). The control law proposed in this example is  $u_k = -Kx_{k-d_{2_k}}$ 

where

$$K = [-0.2392 - 0.1842]$$

The input delay  $d_1$  is assumed to be time-invariant and the output measurement delay  $d_{2_k}$  is assumed timevarying. The total delay  $d_k = d_1 + d_{2_k}$  ranges between  $d_m = 3$  and  $d_M = 31$ , following the pattern  $d_k =$ 9,11,16,9,27,8,...,9,15,12,29,15.

By applying the LMI constraints proposed in (Gao *et al.* (2008)) (in Theorem 1), stability can be proven for all  $d_M \leq 7$ .

 $<sup>^1\,</sup>$  Notice that the dimension of the set of LMIs to evaluate does not depend on the delay considered but on the order of the system to evaluate.

If we introduce the proposed predictor-control scheme with h = 3, by applying Theorem 1 the closed-loop stability can be assured for all  $d_M \leq 31$ .

The selected value for the delay predictor have been chosen in this example h = 3 since with this value we have obtained a feasible solution guaranteeing the closed-loop stability until  $d_M = 31$ .

In Figures 1 and 2 the responses are depicted for the process state. The initial conditions assumed in the simulation are  $x_1(0) = 1$  and  $x_2(0) = 2$ .

The simulations results show the improvement introduced by the predictor if there is a random time-variant delay as proposed in this example.

## 4. ROBUST-STABILITY OF THE CLOSED-LOOP SYSTEM

Robustness of the designed control to small variations on the time elapsed between sampling instants is analyzed in this section. The study also takes into account the maximum delay that fulfills Theorem 1, i.e. when  $d = d_M$ . Afterwards, the analysis is extended to the case where there are small uncertainties in the nominal matrices of the plant model.

Let us define  $t_k$  as the k-th sampling instant, such that

$$T_k = t_{k+1} - t_k + \zeta_k$$

where  $\zeta_k$  is a small variation of the time elapsed between sampling instants, and can be positive or negative but always bounded as follows  $|\zeta_k| \leq \overline{\zeta} \ll T$ .

A similar procedure as used in (Lozano *et al.* (2004)) will be here employed. Then, (8) can be stated as

$$x_{k+1} = (A - BK)x_k - BKA^h x_{k-h} + BKA^h x_{k-d} + \Gamma' \gamma_k$$
  
being the control input

$$u_{k-d} = -K[x_k + A^h x_{k-h} - A^h x_{k-d}] + \Gamma'' \gamma_k$$

where  $\Gamma'$  and  $\Gamma''$  are matrices whose elements are bounded by  $\overline{\zeta}$  (consequently,  $\Gamma'$  and  $\Gamma''$  converge to zero as  $\overline{\zeta}$  goes



Fig. 1. Comparison between nominal case and time-variant delayed case for system state  $x_1$ 



Fig. 2. Comparison between nominal case and time-variant delayed case for system state  $x_2$ 

to zero), and  $\gamma_k$  the extended vector that collects the next and current state, as well as the past states and the delayed inputs, i.e.

$$\gamma_k = [x_k, \cdots, x_{k-h}, \cdots, x_{k-d}, u_{k-d-1}, \cdots, u_{k-2d-1}]$$

With obvious notation, the closed-loop system can be written as

$$\gamma_{k+1} = \bar{A}\gamma_k + \Gamma\gamma_k \tag{19}$$

It can be seen from (19) that, if d = h the eigenvalues of  $\overline{A}$  are given by the set of the *n* eigenvalues of (A - BK) and ((d+1)(n+m) - n) eigenvalues at the origin, due to the introduced delays. On the other hand, if  $d \neq h$ , but the LMI constrain (15) is fulfilled,  $\overline{A}$  is also a Schur matrix, i.e.  $\overline{A}$  has all its eigenvalues strictly inside the unit circle. Thus, it follows that, for every  $Q > 0 \exists P > 0$  such that the following Lyapunov equation holds

$$\bar{A}^T P \bar{A} - P = -Q.$$

Then, other than the time delay, if uncertainties in the plant model,  $A_c$ ,  $B_c$ , and  $\zeta_k$  are small enough such that

$$-Q + \|2\tilde{\Gamma}^T P\bar{A} + \tilde{\Gamma}^T P\tilde{\Gamma}\| < -\eta Q$$

for some  $\eta > 0$ , the closed-loop system (19) remains stable.

#### 5. EXPERIMENTAL RESULTS

A 3D helicopter, built by Quanser (Quanser (Online)), has been used to validate the proposed control scheme.

The 3D Hover system consists of a frame with four propellers mounted on a 3 DOF pivot joint such that the body can freely roll, pitch and yaw. The propellers generate a lift force that can be used to control the pitch and roll angles. The total torque generated by the propeller motors causes a yaw to the body as well. Two propellers in the system are counter-rotating propellers such that the total torque in the system is balanced when the thrusts of the four propellers are approximately equal.

All electrical signals to and from the body are transmitted via a slipring thus eliminating the possibility of tangled wires and reducing the amount of friction and loading about the moving axes, more details see (Quanser (Online)). The three rotations are measured using optical encoders.

The angular dynamics of the helicopter, assuming small angles, can be expressed as (see Castillo *et al.* (2005))

$$\ddot{\eta} = \bar{K}_{\eta} u_{\eta} \qquad \forall \eta = \psi, \theta, \phi$$

where  $\eta = \psi, \theta, \phi$  are the Euler angles (yaw, pitch, and roll respectively),  $u_{\eta}$  is the angular control input and  $\bar{K}_{\eta}$  is a constant that contains the inertia moment of the body and a constant gain that is proportional to the relation between force and voltage. The control objectify is to stabilize, in a desired position, only the roll angle of the helicopter, that is, we assume fixed pitch and yaw angles. Thence, it follows

$$\ddot{\phi} = \bar{K}_{\phi} u_{\phi} = \bar{K}_{\phi} (V_r - V_l)$$

where  $\bar{K}_{\phi} = 0.4235$  and  $V_r, V_l$  are the right and left voltages of the motors respectively. Define  $x = [\phi \ \dot{\phi}]'$ , thus, the above yields,

$$\dot{x}(t) = A_c x(t) + B_c u(t)$$

with

$$A_c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad B_c = \begin{pmatrix} 0 & 0 \\ \bar{K}_{\phi} & -\bar{K}_{\phi} \end{pmatrix}; \quad u(t) = \begin{pmatrix} V_r \\ V_l \end{pmatrix}$$

To stabilize the above system, it is proposed the follows control law

$$u(t) = -\begin{pmatrix} 136.5 & 45.3 \\ -136.5 & -45.3 \end{pmatrix} \hat{x}(k) + K_r \int (y(t) - r(t)) \quad (20)$$

where  $\hat{x}$  is defined as  $\hat{x} = [\phi \ \dot{\phi}]'$  and  $K_r = 0.6$  is a constant. Remember that, the sensor only gives the angular orientation and by consequence, the angular rate can not be directly measured. However, a reduced order observer with the following form is designed to estimate this state (Quanser (Online)),

$$\hat{\phi}(t) = \frac{125.7^2 s}{s^2 + 150.8s + 125.7^2} \tag{21}$$

To improve the experiments, we propose like a desired position a square wave function. Figure 3 illustrates the well performance of the controller in the ideal case, i.e. without delay.

Afterwards, some virtual delays, in the input channel  $u_k$  (time-constant delay  $\tau_1$ ) and in the output channel  $y_k$  (random time-varying delay  $\tau_{2_k}$ ), have been introduced to the system.

In order to know the maximum delay supported by the system before to be unstable, we change on-line, in the experiments, the constant delay, for this,  $\tau_1$  yields,

$$\tau_1 = dT + \Delta_i$$

In the application, the following values are used: d = 80, T = 1 ms,  $\Delta_i = 0, 10, 20, 30 \text{ ms}$  and  $T \leq \tau_{2_k} \leq 20T$ . Figure 4 shows the system response when adding virtual delays. Notice that the system becomes unstable for a delay  $\tau_1 \geq 0.110s$ .

The implementation of the predictor (6) is doing with the following values:

• the sampling period considered is T = 1 ms,



Fig. 3. Closed-loop system response without delay (nominal case) using a square wave function as a desired angle.

- h is computed using the average value of  $\tau_{2_k}$ , that is 0.01 plus the value of  $\tau_1$  that yields the system unstable. Then the nominal value is h = 120,
- the matrices A and B are the discretization of  $A_c$  and  $B_c$  respectively.

With the aim to prove robustness against uncertainties in the delay, we have considered different input delay. Figure 5 shows clearly the improvement achieved by the predictor scheme but also the robustness of the proposed prediction scheme even in presence of uncertainties in the input delay. In addition, the obtained closed-loop system behavior is very similar to delay-free case.

This result shows that the predictor scheme could be pursued to restore the desired closed-loop behavior, even when time-varying delay-range is subject to some uncertainty.



Fig. 4. Closed-loop system response without predictor, time-varying output delay  $T \leq \tau_{2_k} \leq 20T$ , and input delay  $\tau_1 = dT + \Delta_i$ . The arrows indicate when the input delay is modified.



Fig. 5. Closed-loop system response with predictor (h = 120) and time-varying output delay  $T \le \tau_{2_k} \le 20T$ . Initially the input delay is  $\tau_1 = 0.10$ s. The arrows indicate when the input delay is modified.

#### 6. CONCLUSIONS

In this paper a discrete-time state prediction based control scheme for stabilizing unstable continuous-time delay systems has been analyzed. This control scheme has been shown to be robust with respect to time-varying delays in the sensor output channel, besides the possible parametric model uncertainties. A sufficient condition for timevarying delay-dependent stability has been provided when the errors in the delay are larger than the sampling period.

The prediction control strategy has been validated in a quad-rotor helicopter. Real-time experiments have enlighten the performance of the prediction based controller and have satisfactorily demonstrated its robustness with respect to plant delay errors larger than the sampling period and the presence of random variations at sensor output delay. Also, unavoidable measurement noise has not been a major problem.

One of the main contributions of this work concerns the digital implementation and the experimental validation of the proposed algorithm in a real prototype. Note that, this prototype is unstable with very fast dynamics where the lost of one sample could be critical.

#### REFERENCES

- Astrom K.J. and B. Wittenmark Computer-Controlled Systems Theory and Design *Pretince-Hall, third edition*,1997.
- Boyd, S. and Ghaoui, L.E. and Feron, E. and Balakrishnan, V. Linear Matrix Inequalities in System and Control Theory SIAM Studies in Applied Mathematics, Vol. 15, 1994.
- Castillo P., Lozano R., and Dzul A., Modelling and control of mini flying machines *Springer-Verlag*, 2005.
- Gu, K. and Niculescu, S.I. Survey on Recent Results in the Stability and Control of Time-Delays Systems Journal of Dynamic Systems, Measurement, and Control, Vol. 125, 2003, pp 158-165.

- H. Gao, J. Lam, C. Wang and Y. Wang Delay-dependent output-feedback stabilisation of discrete-time systems with time-varying state delay *IEE Proc.-Control Theory Appl.*, Vol. 151, No. 6, November 2004.
- Garcia, P. Castillo, P. Lozano, R. and Albertos, P. Robustness with respect to delay uncertainties of a predictorobserver based discrete-time controller in Proc. of the 45th IEEE Conference on Decision and Control, 2006.
- Gudin R. Robust control using dead-time Faculty compensators, Master's thesis. of Me-Eng., chanical TechnionIIT, Nov. 2007[Online] http://library.technion.ac.il/thesis/mec/2292113.pdf.
- Lozano, R. Castillo, P. Garcia, P. Dzul, A. Robust prediction-based control for unstable delay systems: Application to the yaw control of a mini-helicopter *Automatica*, Vol. 40, 2004, pp 603-612.
- Pan, Y-J. Marquez, H.J, Chen, T. Remote Stabilization of Networked Control Systems with Unknown Time Varying Delays by LMI Techiques 44th IEEE Conference on Decision and Control and the European Control Conference, Seville, Spain, Decembre 2005, pp. 1589-1594.
- Manitius, A. Z. and Olbrot, A.W. Finite spectrum Assignment problem for Systems with Delays *IEEE Transactions on Automatic Control*, AC-24 (4), 1979, pp 541-553.
- Mirkin, L. Are distributed-delay control laws intrinsically unapproximable? 4th IFAC Workshop on Time-Delay Systems (TDS'03), Rocquencourt, France 2003.
- Mirkin, L. On the approximation of distibuted-delay control laws Systems and Control Letters, Vol. 51, 2004, pp. 331-3422.
- Mondie, S., Dambrine, M., and Santos, O. Approximation of control laws with distributed delays: a necessary condition for stability *IFAC Conference on Systems*, *Structure and Control*, *Praque*, *Czek Republic*, 2001.
- Mondie, S., Lozano, R., and Collado, J. Reseting processmodel control for unstable systems with delays 40th Conference Decision and Control, Orlando, 2001.
- Mondie, S. and Michiels, W. Finite spectrum assignment of unstable time-delay systems with a safe implementation *IEEE Trans. Automat. Contr.*, Vol. 48, 2003, pp. 2207-2212.
- Palmor, Z.J. Time delay Compensation-Smith predictor and its modifications The Control Handbook (W.S. Levine, Ed.), pp. 224-237, CRC Press, 1996.
- Quanser Manual, Quanser, 3D\_Hover System: Manual [Online] http://www.quanser.com/english/downloads/ products/3DOF\_Hover.pdf.
- Richard, J.P. Time-delay systems: an overview of some recent advances and open problems *Automatica*, Vol. 39, 2003.
- Smith, O.J.M. Closer Control of loops with dead time *Chemical Engineering Progress*, Vol. 53, 1959, pp 217-219.
- Seron, M., Braslavsky J., and Goodwin G. Fundamental Limitations in Filtering and Control [Online] http://www.eng.newcastle.edu.au / jhb519/book/download.html
- Zhong, Q.C On Distributed Delay in Linear Control Laws-Part I: Dicrete-Delay Implementations IEEE Trans. Automat. Contr., Vol. 49, 2004, pp. 2074-2080.