# Smith Predictor-Based Control Schemes for Dead-Time Unstable Cascade Processes

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This paper presents two simple and efficient Smith predictor (SP) based control schemes which can be used to control open-loop stable or unstable time-delay cascade processes. The proposed structures have two control loops, a secondary inner loop and a primary outer loop. Similar to previous approaches, the secondary loop uses an internal model control (IMC) structure. Two different schemes are proposed for the outer loop that has an unstable open-loop behavior. Contrary to previous proposed controllers, where a delayed model should be used in the stabilization and tuning procedure by considering some kind of polynomial approximation of the dead time, in the proposed structures, internal stability is naturally achieved through a suitable implementation and tuning of the controller without using any delay approximation. To illustrate this, simulation comparative results with some of the schemes recently presented in the literature are presented, showing the simplicity of the proposed design. Moreover, the simulations show that the proposed schemes allow one to obtain some improvement in disturbance rejections performance.

#### 1. Introduction

Many processes in industry, as well as in other areas, exhibit dead times in their dynamic behavior. Conventional controllers, such as PID controllers, could be used when the dead time is small, but they show poor performance when the process exhibits long dead times. In these cases, it is convenient to introduce a dead-time compensating (DTC) structure. <sup>2,3</sup>

The Smith predictor<sup>4</sup> (SP), and its many extensions, was used to improve the performance of classical controllers for stable plants with dead time. However, for open-loop unstable dead-time processes, the original SP is unstable.<sup>1,5</sup> Over recent years, numerous extensions and modifications of the SP have been proposed in order to allow its use with unstable plants.<sup>6–11</sup>

Controlling processes with long time delays and subjected to strong disturbances with the standard feedback control loop sometimes does not result in good enough performance.<sup>12</sup> Cascade control<sup>13</sup> is one strategy that can be used to improve disturbance rejection performance in several situations. The idea of cascade structure is that the effect of the disturbance on the main controlled variable is reduced by an internal (or secondary) loop when an intermediate measurement is available. Cascade control loops are normally used in the process industry for control of temperature, flow, and pressure loops.<sup>14</sup>

Control strategies that combine cascade control with deadtime compensation structures are interesting solutions to control unstable processes with significant dead times and subjected to strong disturbances in the inner loop. Because of this, in the past few years this subject has attracted the attention of several researchers. <sup>15–18</sup> In refs 15 and 16 the proposed methods do not consider systems with zeros; the control structure involves many controllers, and the design methods are difficult to be used. <sup>17</sup> To overcome these problems Uma et al. <sup>17,18</sup> proposed a new modified Smith predictor combined with cascade control to control processes with an unstable or an integrative mode. The proposed scheme shows basically three improvements over previous strategies: (i) it considers processes with a zero; (ii) it uses only three controllers in the principal loop; (iii) the tuning procedure is easier. However, neither this controller nor the previous ones fulfill the Smith philosophy, that is, the design of all the necessary controllers is done without considering any delay. Because of this, the delay is not removed from all the sensitivity functions and they need a polynomial approximation of the delay in the tuning procedure or analog controller implementation. Moreover, the design of the mentioned proposals is limited to specific low-order processes (most of them are only for first-order models), and none of them consider the discrete implementation of the control law.

In this paper, two equivalent structures based on the SP philosophy to control unstable time delay cascade systems are presented. In the proposed strategies the focus is on the design and tuning of the controller for the unstable dead-time system of the principal loop; thus, a conventional IMC structure<sup>19</sup> is used to control the inner loop, as in refs 15-17. The proposed structures are based on two recently published controllers<sup>20,21</sup> and have some advantages over other structures: (i) both schemes are much simpler than the ones proposed in previous works<sup>15–18</sup> and give similar or better closed-loop responses; (ii) tuning of primary controllers of the proposed DTC structures is done without considering any delay; (iii) tuning is simple because simple filters are used to improve robustness, disturbance rejection performance, or noise attenuation; (iv) they are designed for the general discrete case, that is, they can be used with any process model order and the implementation is straightforward.

Although the proposed controllers are general, simple models are used in the examples of this paper to allow a comparative analysis with previous solutions. Moreover, it is important to note that, in industry, easy to understand and tune control structures are very important, and on this point the proposed controllers show an important advantage when simple models are considered to describe the process dynamics.

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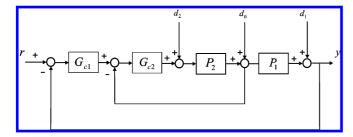


Figure 1. Cascade control system.

The rest of the paper is as follows. The next section presents the cascade control of unstable dead-time systems and revises the last controllers presented in the literature. Section 3 is devoted to introducing the proposed cascade schemes, and some comparative examples are given in section 4. The paper ends with some conclusions.

#### 2. Cascade Control of Unstable Time Delay Processes

In process industry more than set-point tracking disturbance rejection is the principal goal, as most industrial processes operate with a fixed set point during long periods of time. In several situations the process can be modeled by two blocks connected in series, where it is possible to have an auxiliary measurement variable in addition of the controlled one. This idea is depicted in Figure 1, where the first part of the process, modeled by  $P_2(s)$ , has a faster dynamics than the second one, modeled by  $P_1(s)$ , and external disturbances are represented by  $d_0$ ,  $d_1$ , and  $d_2$ . Therefore, the auxiliary measurement variable reflects the effect of the disturbance before it excites the slow dynamics of the process. It is just in these cases where cascade control can be used to improve the disturbance rejection response of the closed-loop system. This type of structure is usually used in flow-level control, flow-temperature control, and speedposition control, among others. 22 The traditional design consist of two steps: first, the internal loop is controlled by means of  $G_{c2}$ , and then  $G_{c1}$  is computed. The design and tuning problem is much more involved if the process has delays. Delays can appear in both primary and secondary processes, but in nature time delays more frequently appear in the primary process, which is the slowest one. Because of the dead time, it is difficult to tune standard feedback controllers for these systems, mainly when fast closed-loop responses are required. In these cases, a DTC-based scheme, as conventional SP or IMC strategies, can be used to improve the performance obtained with standard controllers. 12

However, if the primary process is unstable, the original SP or IMC schemes cannot be directly used because they are not internally stable; thus, a specific DTC-based control scheme which avoids internal instability is sought. This problem has attracted the attention of the control community in recent years, and recently, some control structures have been proposed to avoid the internal instability for open-loop unstable dead-time cascade processes. 15-17 A brief review of theses strategies is presented in the following to point out their advantages and drawbacks and to motivate the presentation of section 3, where two new solutions are proposed. Four main aspects of the revised controllers are highlighted in this section: the process model order, the number of controllers to be tuned, the design and implementation complexity, and the fulfillment (or not) of the Smith principle, that is, the design of all controllers is made without considering any delays.

**2.1.** Liu et al.'s Controller. In Liu et al.<sup>15</sup> the scheme in Figure 2 is proposed for cascade control of open-loop unstable

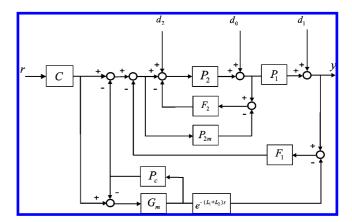


Figure 2. Liu et al.'s cascade control structure.

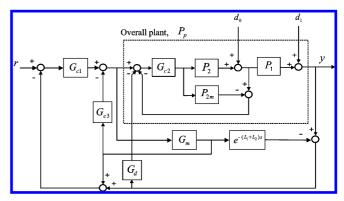


Figure 3. Scheme proposed by Kaya et al., 2008.

processes with delays, where  $P_1$  and  $P_2$  represent the two parts of the delayed process and  $L_1$  and  $L_2$  the corresponding delays. As can be seen, an IMC structure is used and four controllers must be tuned in this scheme:  $F_2$ ,  $F_1$ ,  $P_c$ , C. In the nominal case, that is, if there are not modeling errors, from the scheme in Figure 2 the following closed-loop transfer functions are obtained

$$\begin{split} H_{yr} &= \frac{CP_{2m}P_{1m}}{1 + P_cG_m}; \, H_{yd_2} = \frac{P_{2m}}{1 + F_2P_{2m}} \\ H_{yd_0} &= \frac{1}{1 + F_2P_{2m}}; \, H_{yd_1} = \frac{1}{1 + F_1P_{2m}P_{1m}} \end{split}$$

where  $P_{1m}$  is the model of  $P_1$ ,  $P_{2m}$  is the model of  $P_2$ , and  $G_m$  is the delay-free model of  $P_1P_2$ . From the above nominal expressions it can be noted that controllers C and  $P_c$  can be determined using the delay-free part of the overall plant transfer function model, but this is not the case for  $F_1$  and  $F_2$ . In particular, the design of  $F_2$  uses a polynomial approximation of the obtained nonrational expression of  $F_2$  based on a Maclaurin expansion formula or a Padé series expansion. Moreover, the design procedure is limited to a first-order plus dead time (FOPDT) and an unstable first-order plus dead time (UFOPDT) for the secondary and primary process models, respectively.  $^{15}$ 

**2.2. Kaya et al.'s Controller.** An SP-based controller was presented in Kaya et al.<sup>16</sup> based on the structure of Figure 3. Also, in this case, four controllers must be tuned:  $G_{c1}$ ,  $G_{c2}$ ,  $G_{c3}$ ,  $G_d$ . Similarly to the previous case,  $P_{2m}$  is the model of  $P_2$ ,  $P_{1m}$  is the model of  $P_1$ , and  $P_2$  is the delay-free model of  $P_1$ .

In the nominal case, from the scheme in Figure 3, the following sensitivity functions are used to tune the controller

$$H_{yr} = \frac{G_m G_{c1} e^{-(L_1 + L_2)s}}{1 + G_m [G_{c1} + G_{c3}]}$$

$$H_{yd_0} = \frac{P_1 (1 - G_{c2} P_{2m})}{[1 + G_m (G_{c1} + G_{c3})]} \times \frac{[1 + G_m G_{c3} + G_m G_{c1} (e^{-(L_1 + L_2)s} - 1)]}{(1 + G_d G_m e^{-(L_1 + L_2)s})}$$

where  $G_m e^{-(L_1 + L_2)s} = P_{1m} M_{imc}$  and  $M_{imc}$  is the closed-loop transfer function of the internal loop. In the nominal case  $M_{imc} = G_{c2} P_2$ .

These expressions revel that, in the nominal case, the controllers  $G_{c1}$ ,  $G_{c2}$ , and  $G_{c3}$  can be determined using the delay-free part of the overall plant transfer function model, but the tuning of  $G_d$  is delay dependent. As in the previous analyzed controller, the design methodology presented in this paper is restricted to two special cases: UFOPDT or integral plus dead-time (IPDT) process models. <sup>16</sup>

**2.3.** Uma et al.'s Controller. Recently Uma et al.<sup>17</sup> proposed the scheme shown in Figure 4 with, again, four controllers:  $G_{cs}$ ,  $G_{c2}$ ,  $G_{cd}$ ,  $G_f$ . Although in this new controller the idea is similar to the previous one, the tuning is simpler and it can cope with processes with zeros.  $G_f$  is a predictor error first-order filter, and the other three controllers have to be tuned to obtain the desired closed-loop responses. Although in ref 17 the authors showed that significant improvement is obtained when this strategy is compared to previous reported methods in the literature, the approach only considers first-order models with a zero. Again, in the ideal case, from scheme in Figure 4, the closed-loop relationships are

$$H_{yr} = \frac{G_{cs}G_m e^{-(L_1 + L_2)s}}{1 + G_{cs}G_m}$$

$$H_{yd_2} = \frac{P_1 P_2 (1 + G_m G_{cs} - G_f G_{cs} G_m e^{-(L_1 + L_2)s})}{(1 + G_m G_{cs})(1 + G_{cd} G_m e^{-(L_1 + L_2)s})}$$

$$H_{yd_0} = \frac{P_1 (1 + G_m G_{cs} - G_f G_{cs} G_m e^{-(L_1 + L_2)s})}{(1 + G_m G_{cs})(1 + G_{cd} G_m e^{-(L_1 + L_2)s})}$$

where  $G_m e^{-(L_1 + L_2)s} = P_{1m} M_{imc}$  and  $M_{imc}$  is the closed-loop transfer function of the secondary loop. As can be seen from these expressions, the tuning of  $G_{cd}$  is delay dependent and this is solved by using a first-order Padé approximation of the time delay.<sup>17</sup>

The main conclusions of the analysis of these controllers (which are those reporting the best results for cascade control of unstable dead-time processes) are as follows: (i) all structures use more than three controllers in the design; (ii) they are restricted to simple process models; (iii) they do not verify the SP principle for all the involved characteristic equations; (iv) the overall structure and tuning rules are not easy to understand and use; (v) most of them use an approximation of the delay to obtain at least one of the controllers, and (vi) digital implementation issues are not addressed, which are fundamental for practical applications. Thus, to give a solution for the cited drawbacks of the analyzed schemes, two new cascade control structures are proposed in the next section for unstable dead-time processes.

# 3. Proposed Schemes Based on the Smith Predictor Philosophy

In this section, two new output prediction-based cascade control schemes are proposed to control unstable time delay

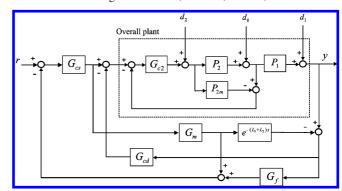


Figure 4. Scheme proposed by Uma et al., 2009

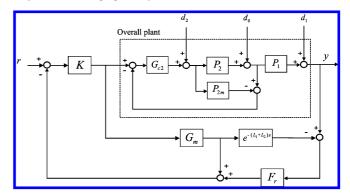


Figure 5. FSPCC analysis structure.

systems: the filtered Smith predictor cascade control (FSPCC) and the generalize predictor cascade control (GPCC). These controllers are closely related mainly because (i) they are directly defined in the discrete domain and (ii) internal stability is achieved by eliminating the unstable poles from the predictor structure, using an explicit procedure in the FSPCC and an implicit one in the GPCC. As will be shown later, the proposed schemes solve the drawbacks of the controllers analyzed in the previous section.

**3.1. Filtered Smith Predictor Cascade Controller.** The FSPCC is a DTC that can be used to control stable, unstable, and integrative dead-time processes in a cascade configuration with a unified tuning approach. The conceptual structure of the FSPCC is depicted in Figure 5, where  $P_{2m}$  is the model of  $P_2$ ,  $G_{c2}$  is the internal loop IMC controller, K is the external loop controller,  $F_r$  is the prediction filter (also called robustness filter), and  $P_m = G_m e^{-(L_1 + L_2)s}$  is the overall plant model which includes the internal loop (see Figure 5).

The FSPCC is derived from the FSP strategy, in which it is possible to deal with robustness and disturbance rejection aspects by means of a filter  $F_r$  that does not change the nominal setpoint response. <sup>1,11</sup> In this controller, an IMC internal loop is used to compare the results with previous works; however, any other controller can be defined for this internal loop.

As the final control law is implemented in discrete time, from now on a discrete equivalent system is used to analyze the design and tuning of the controller (the same procedure is used in the GPCC in the next section). This equivalent system is obtained using traditional discretization tools with a sampling time T defined using the procedure suggested in ref 1 (Chapter 8). Thus, the implementation structure of the FSPCC is shown in Figure 6 where  $S(z) = G_m(z)(1 - z^{-h}F_r(z))$  and  $F_r(z)$  and K(z) are the controllers of the principal loop. Here, h represents the overall delay, in samples, of both loops.  $F_r(z)$  is used to guarantee that S(z) is stable, avoiding internal instability problems when there are unstable or integrative poles in  $P_m(z)$ , and to give a

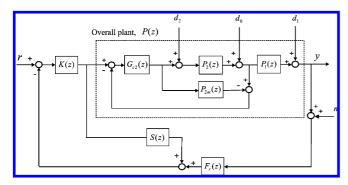


Figure 6. FSPCC implementation structure.

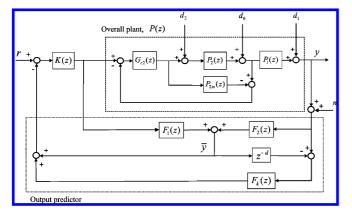


Figure 7. Scheme of the GPCC.

compromise between robustness and disturbance rejection, as it is analyzed in the following sections.

**3.2. Generalized Predictor Cascade Controller.** The GPCC is a strategy specially oriented to control unstable and integrative dead-time processes in a cascade configuration. Its structure is depicted in Figure 7.

Let us define the undelayed system output such as

$$\bar{y}(z) \doteq G_m(z)u(z) = G_n(z)\Gamma(z)u(z)$$
 (1)

where  $G_m$  is the delay-free model of the overall process, that is,  $P(z) = G_m(z)z^{-h}$ , and  $\Gamma(z)$  is a polynomial representing its all nonminimal phase zeros and those stable zeros (nonminimal phase zeros and stable zeros are, respectively, zeros outside and inside the unitary circle) located close to the unitary circle which can cause measurement noise amplification in the control action. Note that for systems without this type of zeros,  $\Gamma(z) = 1$  and  $G_m = G_n$ .

Then, the undelayed system output (eq 1) can be computed as (see ref 20 for details)

$$\bar{y}(z) = F_1(z)u(z) + F_2(z)y(z)$$
 (2)

where  $F_1$  and  $F_2$  are stable filters being defined

$$F_1(z) = c \sum_{i=1}^{h} A^{i-1} b z^{-i} \Gamma(z)$$
 (3)

$$F_2(z) = \frac{G_n^*(z)}{G_n(z)} = \frac{c(zI - A)^{-1}A^hb}{c(zI - A)^{-1}b}$$
(4)

and (A, b, c) is a minimal state space representation of  $G_n(z)$ . In this strategy the prediction error filter  $F_k(z)$  can be used to improve robustness or noise attenuation. This filter must have unitary static gain  $(F_k(1) = 1)$  in order to reject step disturbances.

**3.3. Internal Stability and Robust Stability.** Consider first the structure of the FSPCC. In the ideal case, i.e., when there are no uncertainties, from the scheme in Figure 6, the following expressions can be obtained (for simplicity, the dependence with *z* is omitted in the following)

$$H_{yr}(z) = \frac{P_m K}{1 + KG_m} \tag{5}$$

$$H_{yd_2}(z) = \frac{(1 + KS)P_1P_2}{1 + KG_m}(1 - M_{imc})$$
 (6)

$$H_{yd_0}(z) = \frac{(1 + KS)P_1}{1 + KG_m}(1 - M_{imc}) \tag{7}$$

$$H_{yd_1}(z) = \frac{(1 + KS)}{1 + KG_m} \tag{8}$$

$$H_{yn}(z) = -\frac{F_r K P_m}{1 + K G_m} \tag{9}$$

where  $M_{imc}$  is the stable desired closed-loop transfer function of the IMC internal loop (obtained with  $G_{c2} = M_{imc}/P_2$ ) and the overall plant model is  $P_m = M_{imc}P_1 = G_mz^{-h}$ . Thus, if K(z) is computed to obtain a stable  $H_{yr}(z)$  (solving  $1 + K(z)G_m(z) = 0$ , that is, a delay-free stabilization problem) also  $H_{yn}(z)$  and  $H_{yd_1}(z)$  are stable. To show that  $H_{yd_2}(z)$  and  $H_{yd_0}(z)$  are also stable it is sufficient to note that  $G_m(z)$  and  $P_m(z)$  have the same poles and that S(z) is stable; thus, the internal stability is guaranteed as K stabilizes the main loop.

To show how  $F_r(z)$  is selected for internal stability, that is, to have a stable S(z), consider that  $F_r(z) = N_r(z)/D_r(z)$ ,  $F_r(1) = 1$ , and  $G_m(z) = N_m(z)/D_m^+(z)D_m^-(z)$ , where  $D_m^-$  has all its roots inside the unitary circle and  $D_m^+$  has all its roots with  $|z| \ge 1$ . Thus

$$S(z) = G_m(z)z^{-h}(z^h - F_r(z)) = \frac{N_m(z)z^{-h}}{D_m^+(z)D_m^-(z)} \frac{z^h D_r(z) - N_r(z)}{D_r(z)}$$

Thus, using an arbitrary  $D_r$ ,  $N_r$  is computed to satisfy the following diophantine equation

$$z^{h}D_{r}(z) - N_{r}(z) = D_{m}^{+}(z)(z - 1)p(z)$$
 (10)

where p(z) is an unknown polynomial. Note that contrary to the previous solutions analyzed in section 2, this problem has an exact solution for any dead time and any process model order. Also note that, as expected, the order of the filter depends on the number of unstable roots.

Now consider the GPCC, as in Figure 7. With the same IMC internal loop control used before, the following transfer functions are obtained

$$H_{yr}(z) = \frac{KP_m}{1 + KG_m} \tag{11}$$

$$H_{yd_2}(z) = \frac{G^* P_1 P_2}{1 + KG_m} (1 - M_{imc})$$
 (12)

$$H_{yd_0}(z) = \frac{G^* P_1}{1 + KG_m} (1 - M_{imc})$$
 (13)

$$H_{yd_1}(z) = \frac{G^*}{1 + KG_m} \tag{14}$$

$$H_{yn}(z) = -F_r \frac{KP_m}{1 + KG_m}$$
 (15)

where  $G^* = (1 - KF_kF_1z^{-h} + KF_1)$ ,  $F_r = F_2 + F_k[1 - z^{-h}F_2]$ , and  $M_{imc} = P_2R_{imc}/(1 + R_{imc}P_2)$ .

To prove the internal stability in the GPCC, first note that, as in the FSPCC, K and  $G_{c2}$  are computed to obtain a stable closed-loop control system, that is, K and  $R_{imc}$  are designed to stabilize  $(1 + KG_m)$  and  $(1 + R_{imc}P_2)$ , respectively. Moreover, as  $G_m(z)$  and  $P_1(z)$  have the same unstable poles and the possible unstable poles of  $G^*(z)$  are those in K(z), the internal stability is guaranteed.

The robustness stability analysis could be done in both schemes considering multiplicative uncertainties, that is, assuming a process transfer function  $P_r(z) = P(z)(1 + W_m(z))$ , where  $W_m(z)$  defines the process multiplicative uncertainty term (note that the delay uncertainty can be modeled as multiplicative uncertainty, see refs 1, 19, and 23). In this situation the robust stability condition is obtained using the output-noise sensitivity function<sup>24</sup>

$$||H_{yy}(z)W_{yy}(z)||_{\infty} < 1, z = e^{j\omega T}, \omega \in [0, \pi/T]$$
 (16)

where  $H_{yn}$  is computed for each control strategy. Although conceptually different, if the prediction filters  $F_r$  and  $F_k$  satisfy

$$F_r(z) = F_2(z) + F_k(z)[1 - z^{-h}F_2(z)]$$

it is possible to interpret the GPCC prediction structure (Figure 7) as a particular case of the FSPCC one as it is done in the simple loop DTC case.<sup>25</sup> In this case, both structures, the FSPCC and the GPCC, have the same  $H_{yn}$  and the robust stability analysis can be performed in a unified manner using

$$||F_{r}(z)\frac{K(z)P_{m}(z)}{1+K(z)G_{m}(z)}W_{m}(z)||_{\infty} < 1, z = e^{j\omega T}, \omega \in [0, \pi/T]$$
(17)

Here, it is easy to note that after K(z) was tuned for some setpoint response, a low-pass filter  $F_r(z)$  can be used to improve robustness by selecting an appropriate cutoff frequency. Notice that  $F_r(z)$  does not modify the nominal set-point tracking (eqs 5 and 11), but it affects both the disturbance rejection response and the noise attenuation. Therefore, the tuning of this filter is the crucial point in the design of the FSPCC, and it is detailed in the next section.

**3.4. Robust Tuning Design Procedure.** As pointed out, the controller design starts with the inner loop using a simple IMC tuning which was already deeply explored. <sup>19</sup> For the principal loop the tuning procedure has special steps for the two analyzed controllers. For the FSPCC the tuning can be done in a decoupled manner, that is, first K(z) is tuned for a desired setpoint response  $H_{yr}(z) = H_d(z)z^{-h}$  (note that K can include setpoint weighting tuning parameters); in a second step, if  $F_r = F'_r/H_d(z)$ ,  $F'_r(z)$  allows one to obtain different closed-loop poles

for disturbance rejection and set-point tracking and the internal stability at the same time. Moreover, the degree of freedom of this filter allows one to define the trade off between robustness, disturbance rejection, and noise attenuation, always satisfying the conditions imposed by the unstable model. Note that the order of  $F_r$  increases if the number of control objectives so does. Finally, it must be highlighted that in this controller the digital implementation is straightforward. In the following section, the design procedure is illustrated for several comparative examples.

For the FSPCC and GPCC K is tuned to reach a compromise between robustness and disturbance rejection and in a second step set-point weighting tuning parameters are used to improve the set-point response. If necessary, the predictor error filter  $F_k$  can be included in the GPCC structure in order to improve the disturbance rejection performance.

## 4. Comparative Examples

In this section the proposed schemes are compared with the schemes and tuning presented in refs 16–18, which are recently proposed methods conceived to improve the performance of unstable cascade time-delay systems. For that purpose, the processes and conditions referred in these papers are used. Three examples are considered to illustrate the most representative cases studied in literature: an UFOPDT, a IPDT, and an UFOPDT with a zero system.

As the main objective of this section is to compare the different dead-time compensations strategies, tuning choices for both inner and outer controllers are not a matter of discussion. As a consequence, the controllers used in the proposed schemes, that is,  $G_{c2}(z)$  and K(z), are, respectively, obtained using a polezero matching discretization of the continuous controllers proposed in refs 16-18 in each of the examples.

Moreover, for the sake of tuning and implementation simplicity, when necessary (note that in Example 3  $F_k(z) = 1$ ), the GPCC prediction error filter,  $F_k(z)$ , is also obtained from a polezero matching discretization of  $G_f(s)$ .<sup>17</sup>

In the FSPCC case, differently from the GPCC, all the measured signals are filtered by  $F_r(z)$ , which may slow down disturbance rejection response. Thus, a simple phase lead is used to improve disturbance rejection such as

$$F_r(z) = k_f \frac{(z - \rho_1)(z - \rho_2)}{(z - \gamma_1)(z - \gamma_2)}$$

where  $\gamma_1$  and  $\rho_1$  represents a discrete phase lead,  $\gamma_2$  is a pole mapping of  $G_f(s)$ , and  $k_f$  and  $\rho_2$  should be used to guarantee internal stability as discussed in section 3.3 (internal stability and robust stability). The phase lead parameters  $\gamma_1$  and  $\rho_1$  will be tuned using the phase lead effect of the controller  $G_{cd}(s)$  and will be presented at each example.

As the worst-case model uncertainty depends on the controller tuning, the uncertainties scenarios were extracted from the previous references in order to perform a fair comparison. It is important to emphasize that robustness can be suitably modified without changing the nominal response. As a consequence, simulations will be carried out to illustrate that the proposed strategy outperforms related works in nominal case and leads to similar responses even in the presence of significant plant-model mismatches.

**4.1. Example 1.** Consider the system studied in ref 17. The primary and secondary processes are considered to be



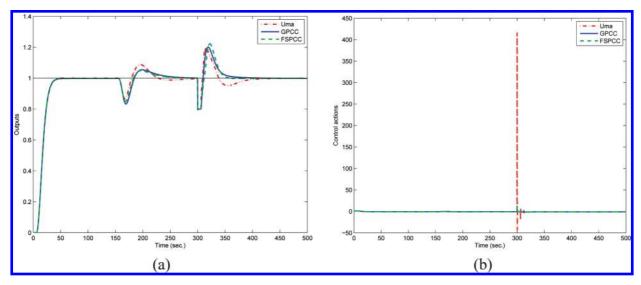


Figure 8. Nominal system responses (process output (a) and control action (b)) for a step load disturbance of -1 at t = 150 s in  $d_2$  and -0.2 at t = 300 in  $d_1$  (Example 1a).

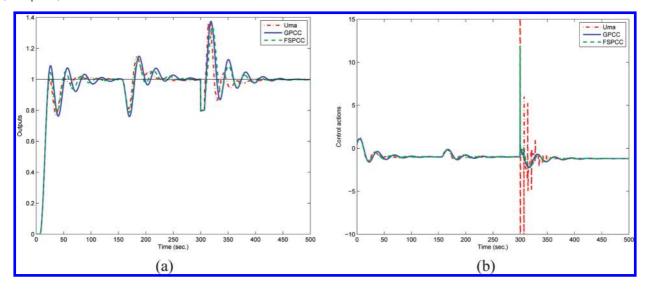


Figure 9. Process output (a) and control action (b) responses for +20% perturbation in the primary time delay and -20% in the primary time constant (Example 1b).

$$G_{p1}(s) = \frac{e^{-4s}}{20s - 1}; G_{p2}(s) = \frac{2e^{-2s}}{20s + 1}$$

In the scheme proposed by Uma et al., <sup>17</sup> three controllers are considered; the inner loop controller  $G_{c2}(s) = (0.5(20s + 1))/(20s + 1)$ (2s + 1), the primary set-point tracking controller  $G_{cs} = (4.6571)$ + 0.1829/s + 12.2857s)(1/(2.8571s + 1)), and the primary disturbance rejection controller  $G_{cd} = (3.1190 + 0.0921/s + 0.0921/s)$ 6.6156s)((3s + 1)/(0.1440s + 1)). The set-point weighting parameter is considered to be  $\varepsilon = 0.3$ , and the prediction error filter is chosen as  $G_t(s) = 1/(36s + 1)$ .

Considering a sampling period of T = 0.2 s, for both GPCC and FSPCC, the following discretized overall free delay model is obtained

$$G_m(z) = \frac{0.00048537(z + 0.9704)}{(z - 1.01)(z - 0.9048)}$$

As pointed out,  $G_{c2}(z)$  and K(z) are, respectively, obtained by using a pole-zero matching discretization of  $G_{c2}(s)$  and  $G_{cs}(s)$ . In the GPCC, to avoid measurement noise amplification, the predicted output, eqs 3 and 4, is computed considering  $\Gamma(z) =$ 

(z + 0.9704)/z and  $G_n(z) = 0.00048537z/[(z - 1.01)(z - 1.01)]/z$ 0.9048)]. For the FSPCC filter, the tuning parameters are  $\gamma_1 = e^{-T/0.144} = 0.2494$ ,  $\rho_1 = e^{-T/3} = 0.9355$ , and  $\gamma_2 = e^{-T/36} = 0.9355$ 0.9945, where 0.144 and 3 correspond to the phase lead parameters of  $G_{cd}$ . Finally,  $\rho_2 = 0.9957$  and  $k_f = 14.9524$  are computed to guarantee that eq 10 holds.

The nominal system responses obtained are shown in Figure 8. As can be seen, process output behavior is similar for the three cases; however, control effort in the scheme proposed by Uma et al. is too high, more than 20 times higher than the control effort of the other two controllers. These high values of the control action are caused by the derivative action of  $G_{cd}$ . Note that FSPCC and GPCC do not use this extra controller and allow for the same performance with a smoother control action.

To show the effect of process uncertainties, perturbations of +20% in the primary time delay and -20% in the primary time constant are considered in the following simulations where step load disturbances of -1 at t = 150 s in  $d_2$  and -0.2 at t = 300in  $d_1$  are considered (as done in ref 17). Figure 9 shows the obtained results. In Figure 9b of this figure the y axis was limited

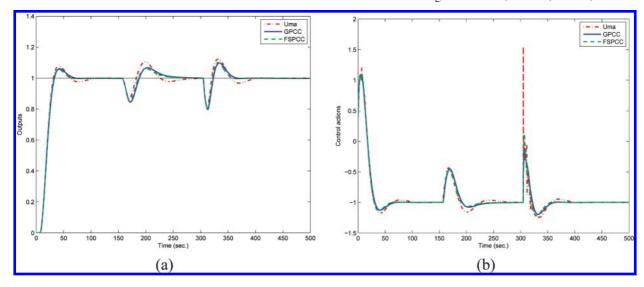


Figure 10. Process output (a) and control action (b) responses for uncertainties of  $\pm 20\%$  in the primary process time delay and time constant. A negative step disturbance in  $d_2$  and  $d_0$  are applied at t = 150 and 300 s, respectively (Example 1c).

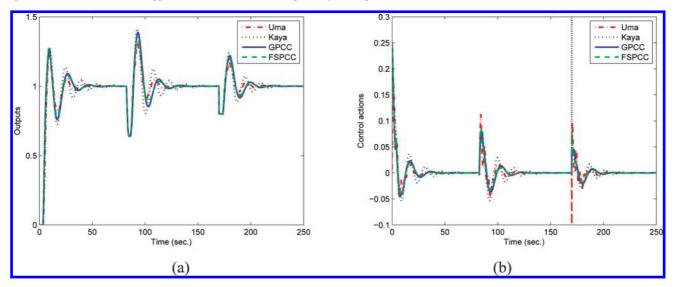


Figure 11. Perturbed system responses (process output (a) and control action (b)) for a step load disturbance of -0.1 at t = 80 s in  $d_0$  and -0.2 at t = 170in  $d_1$  (Example 2a).

to show the details of the control action, as the one obtained with Uma et al.'s scheme achieves values greater than 200 units.

To show the performance for input disturbances in the primary loop ( $d_0$ ), perturbations of +20% in the primary gain, time delay, and time constant are considered (as done in ref 17). Moreover, negative step disturbances in  $d_2$  and  $d_0$  are applied at t = 150 and 300 s, respectively. The obtained responses are shown in Figure 10.

The analysis of the two last cases shows the same situation as in the nominal case; the proposed schemes exhibit slightly better performance with smoother control action.

**4.2. Example 2.** Consider the system previously studied in refs 16 and 18. The primary and secondary processes are considered to be

$$G_{p1}(s) = \frac{2e^{-2s}}{s}; G_{p2}(s) = \frac{4e^{-2s}}{s+1}$$

In the scheme proposed by Uma et al., 18 three controllers are considered; the inner loop controller  $G_{c2}(s) = (0.5(s+1))/(s+1)$ 2), the primary set-point tracking controller  $G_{cs} = 1.8(1 + 1/3.6s)$  + 1.0722s)((0.5s + 1)/(0.6561s<sup>2</sup> + 2.9160s + 1)), and the primary disturbance rejection controller  $G_{cd} = (0.0436 - 0.0436)$ 0.1206s)((0.75 $s^2 + 1.5s + 1$ )/(1.8314 $s^2 + 0.7686s + 1$ )). The set-point weighting parameter is considered to be  $\varepsilon=0.4$ , and the prediction error filter is chosen as  $G_f = 1/(0.6s + 1)$ .

In the scheme proposed by Kaya et al., 16 four controllers are considered (see Figure 3); the inner loop controller  $G_{c2}(s) =$ (0.5(s+1))/(s+2), the primary set-point tracking controller  $G_{c1} = 0.1(1 + 1/4s)$ , and the PD controllers  $G_{c3} = (0.0606 - 1)$ 0.205s) and  $G_d(s) = 0.083(1 + 0.5s)$ .

In the FSPCC and GPCC  $G_{c2}(z)$  and K(z) are, respectively, obtained by using a pole-zero matching discretization of  $G_{c2}(s)$ and  $G_{c1}(s)$  proposed in Kaya et al. In order to obtain the same set-point response as in refs 18 and 16 the following set-point filter is included in both the FSPCC and GPCC

$$F(z) = \frac{(1 - z_0)^2 (1 + G_{c1}G_m)}{(z - z_0)^2 G_{c1}G_m}, \ z_0 = 0.905$$

Using a sampling period T = 0.1 s the discretized overall free delay model is

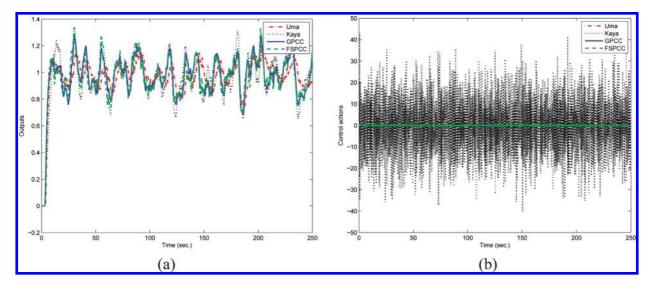


Figure 12. Process output (a) and control action (b) responses for the case with measurement noise (Example 2b).

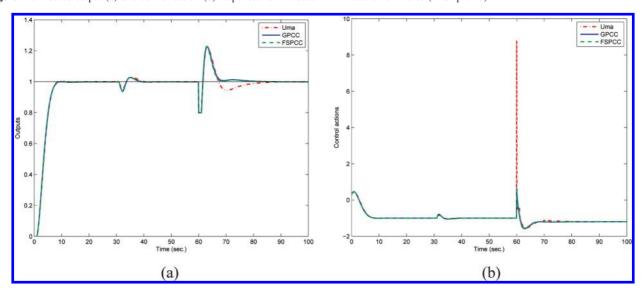


Figure 13. Nominal system responses (process output (a) and control action (b)) for a step load disturbance of -0.05 at t = 30 s in  $d_2$  and -0.2 at t = 60in  $d_1$  (Example 3a).

$$G_m(z) = \frac{0.018731(z + 0.9355)}{(z - 1)(z - 0.8187)}$$

Again, in the GPCC design the prediction output (eqs 3 and 4) is computed considering  $\Gamma(z) = (z + 0.9355)/z$  and  $G_n(z) =$ 0.018731z/[(z - 0.8187)(z - 1)]. In this particular example, filter  $F_r(z) = k_f(z - \rho)/(z - e^{-T})$  with  $\rho = 0.9753$  and  $k_f =$ 3.8549 is computed to guarantee internal stability (eq 10).

In the nominal case, all controllers show similar closed-loop behavior in this example. To analyze the effect of model uncertainties, as it is done in ref 18, perturbations of +20% in the primary time delay and primary process gain are considered in this example. The system responses are shown in Figure 11.

As can be seen, process output behavior is better and control effort is smoother for the FSPCC and GPCC.

Finally, let us consider the effect of a white noise in the measurement device, with a power spectrum of 0.02 (the same as considered in ref 16). The results are shown in Figure 12.

In this case output behavior is similar for all controllers; however, control action in Kaya et al.'s controller is strongly affected by noise, showing high values and high-frequency oscillations.

**4.3.** Example 3. Consider the system previously studied in ref 17. The primary and secondary process are considered to

$$G_{p1}(s) = \frac{(0.1251s + 1)e^{-0.8s}}{(2.828s - 1)}; G_{p2}(s) = \frac{6.1506e^{-0.2s}}{0.3123s + 1}$$

Note that the primary process has a stable zero. In ref 17 it is claimed that the proposed scheme is superior to previous proposals because it can directly deal with this kind of process model.

In the scheme proposed by Uma et al., 17 three controllers are considered; the inner loop controller  $G_{c2}(s) = (0.3123s +$ 1)/(6.150(0.2s + 1)), the primary set-point tracking controller  $G_{cs} = (10.2956 + 2.2534/s + 16.1646s)(1/(3.9841s + 1)),$  and the primary disturbance rejection controller  $G_{cd} = (2.3514 + 1.00)$ 0.2922/s + 0.9845s)((0.5s + 1)/(0.4077s + 1)). The set-point weighting parameter is considered to be  $\varepsilon = 0.3$ , and the prediction error filter is chosen as  $G_f = 1/(6s + 1)$ .

In this example, the sampling period is T = 0.1 s both for GPCC and FSPCC. Then, the discretized overall free delay model is

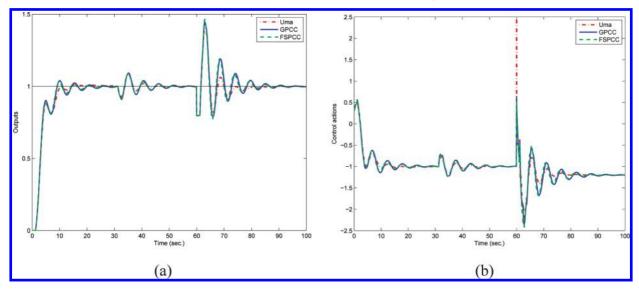


Figure 14. Nominal process output (a) and control action (b) responses for the following modeling errors:  $\pm 20\%$  in both time delays and  $\pm 20\%$  in both the time constants (Example 3b).

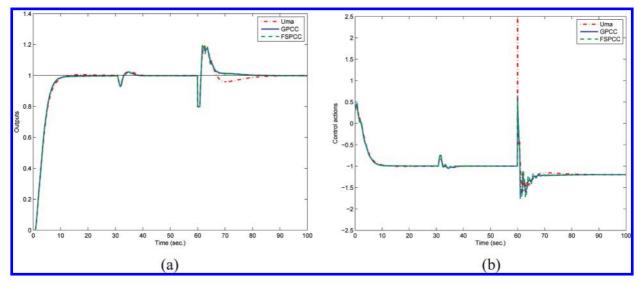


Figure 15. Process output (a) and control action (b) responses for perturbations of -20% in both time delays and time constants (Example 3c).

$$G_m(z) = \frac{0.02537(z - 0.4418)}{(z - 1.036)(z - 0.6065)}$$

 $G_{c2}(z)$  and K(z) are obtained as in previous examples. In this case  $\Gamma(z) = 1$  and  $G_n(z) = G_m(z)$  in the GPCC. In the FSPCC, similarly to the first example, tuning parameters of predictor error filter are  $\gamma_1 = e^{-T/0.4077} = 0.7825$ ,  $\rho_1 = e^{-T/0.5} = 0.8187$ ,  $\gamma_2 = e^{-T/6} = 0.9835$  while  $\rho_2 = 0.9894$ ,  $k_f = 1.8739$  are used to guarantee internal stability. The nominal system responses are shown in Figure 13.

This example shows again the advantages of the FSPCC and GPCC over the controller proposed in ref 17; they need only two controllers and offer better disturbance rejection performance and smoother control action.

To show the effect of uncertainties on the closed-loop performance, different types of perturbations are considered. Figure 14 shows the system responses for +20% perturbations in time delays and -20% in time constants, while Figures 15 and 16 show, respectively, the cases when perturbations of -20% and +20% in both time delays and time constants are considered.

These cases confirm the previous obtained results; FSPCC and GPCC offer a better compromise between control effort and closed-loop behavior.

Finally, let us consider the effect of a white noise in the measurement device, with a power spectrum of 0.002. The results are shown in Figure 17.

Note that although the process output has more or less the same behavior for the three controllers, the effect of noise in the control action is not admissible in the controller proposed in Uma et al.<sup>17</sup> This is again caused by the derivative action of the controller used to obtain an internally stable system. Note that the better trade off between performance and control effort obtained by the FSPCC and GPCC are due to their predictor structure, which do not need this extra controller to obtain an internal stable system.

**4.4. General Remark.** In the schemes proposed in refs 16 and 18 the additional controller used to stabilize the closedloop system  $(G_{cd}(s))$  or  $G_d(s)$  is always a PD controller tuned for stabilization of a delay-dependent loop. Among this disadvantage, which needs some kind of approximation of the delay,

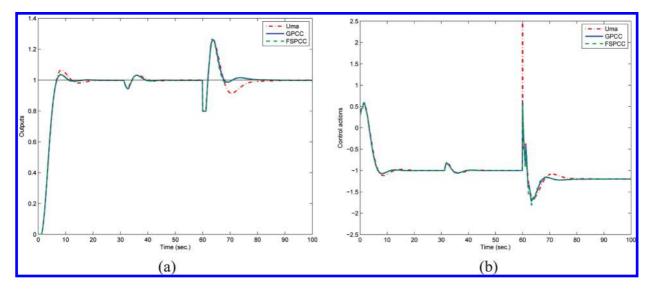


Figure 16. Process output (a) and control action (b) responses for perturbations of +20% in both time delays and time constants (Example 3d).

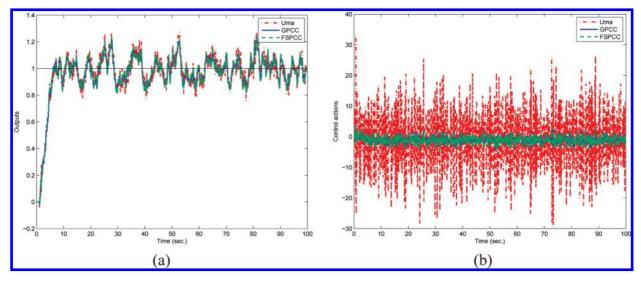


Figure 17. Process output (a) and control action (b) responses for the case with measurement noise (Example 3e).

the implementation of the controller must include a low-pass filter. As has been shown in the examples, the tuning of this filter is not a simple issue. The small time constant of this filter causes strong control action and poor noise attenuation. On the other hand, large values of this time constant can drive the system to instability. Note that FSPCC and GPCC do not need this extra controller; they do not use any delay approximation and do not have these tuning problems.

## 5. Conclusions

This paper presents two simple and efficient dead-time compensator cascade controllers. The proposed strategies have several advantages: (i) they satisfy the Smith principle, (ii) they are formulated in the discrete time domain, thus implementation is straightforward, (iii) they have less controllers to be tuned than in previous proposals, (iv) they are simple to understand and tune, and (v) the tuning considers a trade off between robustness and performance. Moreover, the performance that can be obtained with these two new schemes is always similar or better than the one obtained with previous algorithms: they offer better trade off between performance, control effort, and noise attenuation. This last advantage is due to the predictor structure used in the proposed schemes, which do not need any extra controller to obtain an internal stable system. All these properties and advantages are illustrated in the paper through several comparative simulation examples that consider the most representative cases studied in the literature.

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